The next turn given by the Electrodynamics roadmap for Octonion application to physics requires us to form all expected forces in the vector Action, and the work done in the scalar Action. We must demand this Action Function be an algebraic invariant, since it is a physical observable that must be consistently represented.

Following the classical path, we might expect this form to be the Octonion product of our newly found 8-current and the Octonion field. The 8-current is an algebraic invariant, so it takes on a singular form. The field form however involves a combination of both algebraic variant and invariant. We would be correct to think the product of 8-current and field might also contain both algebraic variant and invariant.

Rather than search out an algebraic invariant form, we could plunge right in with the anticipated form and run it through the Octonion Variance Sieve Process available on my website.

If we run F*j through the sieve process, we get the following

Invariant (F*j)

```
[0]
-j1 [D0(A1)+D1(A0)]
-j2 [D0(A2)+D2(A0)]
-j3 [D0(A3)+D3(A0)]
-j4 [D0(A4)+D4(A0)]
-j5 [D0(A5)+D5(A0)]
-j6 [D0(A6)+D6(A0)]
-j7 [D0(A7)+D7(A0)]

[1]
+j0 [D0(A1)+D1(A0)]
-j2 [D1(A2)-D2(A1)] +j3 [D3(A1)-D1(A3)]
-j4 [D1(A4)-D4(A1)] +j5 [D5(A1)-D1(A5)]
-j7 [D1(A7)-D7(A1)] +j6 [D6(A1)-D1(A6)]

[2]
+j0 [D0(A2)+D2(A0)]
-j3 [D2(A3)-D3(A2)] +j1 [D1(A2)-D2(A1)]
-j4 [D2(A4)-D4(A2)] +j6 [D6(A2)-D2(A6)]
-j5 [D2(A5)-D5(A2)] +j7 [D7(A2)-D2(A7)]

[3]
+j0 [D0(A3)+D3(A0)]
-j1 [D3(A1)-D1(A3)] +j2 [D2(A3)-D3(A2)]
-j4 [D3(A4)-D4(A3)] +j7 [D7(A3)-D3(A7)]
-j6 [D3(A6)-D6(A3)] +j5 [D5(A3)-D3(A5)]

[4]
+j0 [D0(A4)+D4(A0)]
-j5 [D4(A5)-D5(A4)] +j1 [D1(A4)-D4(A1)]
-j6 [D4(A6)-D6(A4)] +j2 [D2(A4)-D4(A2)]
```

-j7 [D4(A7)-D7(A4)] +j3 [D3(A4)-D4(A3)]

[5]
+j0 [D0(A5)+D5(A0)]
-j1 [D5(A1)-D1(A5)] +j4 [D4(A5)-D5(A4)]
-j3 [D5(A3)-D3(A5)] +j6 [D6(A5)-D5(A6)]
-j7 [D5(A7)-D7(A5)] +j2 [D2(A5)-D5(A2)]

[6]
+j0 [D0(A6)+D6(A0)]
-j1 [D6(A1)-D1(A6)] +j7 [D7(A6)-D6(A7)]
-j2 [D6(A2)-D2(A6)] +j4 [D4(A6)-D6(A4)]
-j5 [D6(A5)-D5(A6)] +j3 [D3(A6)-D6(A3)]

[7]
+j0 [D0(A7)+D7(A0)]
-j2 [D7(A2)-D2(A7)] +j5 [D5(A7)-D7(A5)]
-j3 [D7(A3)-D3(A7)] +j4 [D4(A7)-D7(A4)]
-j6 [D7(A6)-D6(A7)] +j1 [D1(A7)-D7(A1)]

These algebraic invariant forms appear to be just what we need. Extracting just the Electrodynamic terms we have

[0]
-j5 [D0(A5)+D5(A0)]
-j6 [D0(A6)+D6(A0)]
-j7 [D0(A7)+D7(A0)]

[5]
+j0 [D0(A5)+D5(A0)]
+j6 [D6(A5)-D5(A6)] - j7 [D5(A7)-D7(A5)]

[6]
+j0 [D0(A6)+D6(A0)]
+j7 [D7(A6)-D6(A7)] - j5 [D6(A5)-D5(A6)]

[7]
+j0 [D0(A7)+D7(A0)]
+j5 [D5(A7)-D7(A5)] - j6 [D7(A6)-D6(A7)]

We can recognize the work -(j dot E) scalar in [0]. The two familiar forces, j0 E and jXB are the vector forms.

The irreducible algebraic variants are shown below. They would need to be assigned values of zero to make the full expression F*j an algebraic invariant.

For minimum distance 1: 1/2[SL(123)+SR(321)]
[4]
-j5 [D7(A6)-D6(A7)] -j6 [D5(A7)-D7(A5)] -j7 [D6(A5)-D5(A6)]
[5]
+j4 [D7(A6)-D6(A7)] +j6 [D4(A7)-D7(A4)] -j7 [D4(A6)-D6(A4)]
[6]
+j4 [D5(A7)-D7(A5)] -j5 [D4(A7)-D7(A4)] +j7 [D4(A5)-D5(A4)]
[7]
+j4 [D6(A5)-D5(A6)] +j5 [D4(A6)-D6(A4)] -j6 [D4(A5)-D5(A4)]

For minimum distance 1: 1/2[SL(123)-SR(321)]
[0]
-j1 [D2(A3)-D3(A2)] -j2 [D3(A1)-D1(A3)] -j3 [D1(A2)-D2(A1)]
[1]
+j0 [D2(A3)-D3(A2)] -j2 [D0(A3)+D3(A0)] +j3 [D0(A2)+D2(A0)]
[2]
+j0 [D3(A1)-D1(A3)] +j1 [D0(A3)+D3(A0)] -j3 [D0(A1)+D1(A0)]
[3]
+j0 [D1(A2)-D2(A1)] -j1 [D0(A2)+D2(A0)] +j2 [D0(A1)+D1(A0)]

For minimum distance 2: 1/2[SL(761)+SR(167)]
[2]
-j3 [D4(A5)-D5(A4)] -j4 [D5(A3)-D3(A5)] -j5 [D3(A4)-D4(A3)]
[3]
+j2 [D4(A5)-D5(A4)] -j4 [D2(A5)-D5(A2)] +j5 [D2(A4)-D4(A2)]
[4]
+j2 [D5(A3)-D3(A5)] +j3 [D2(A5)-D5(A2)] -j5 [D2(A3)-D3(A2)]
[5]
+j2 [D3(A4)-D4(A3)] -j3 [D2(A4)-D4(A2)] +j4 [D2(A3)-D3(A2)]

For minimum distance 2: 1/2[SL(761)-SR(167)]
[0]
-j1 [D7(A6)-D6(A7)] -j6 [D1(A7)-D7(A1)] -j7 [D6(A1)-D1(A6)]
[1]
+j0 [D7(A6)-D6(A7)] +j6 [D0(A7)+D7(A0)] -j7 [D0(A6)+D6(A0)]
[6]
+j0 [D1(A7)-D7(A1)] -j1 [D0(A7)-D7(A0)] +j7 [D0(A1)+D1(A0)]
[7]
+j0 [D6(A1)-D1(A6)] +j1 [D0(A6)+D6(A0)] -j6 [D0(A1)+D1(A0)]

For minimum distance 3: 1/2[SL(572)+SR(275)]
[1]
+j3 [D4(A6)-D6(A4)] -j4 [D3(A6)-D6(A3)] +j6 [D3(A4)-D4(A3)]
[3]
-j1 [D4(A6)-D6(A4)] -j4 [D6(A1)-D1(A6)] -j6 [D1(A4)-D4(A1)]
[4]
+j1 [D3(A6)-D6(A3)] +j3 [D6(A1)-D1(A6)] -j6 [D3(A1)-D1(A3)]
[6]
-j1 [D3(A4)-D4(A3)] +j3 [D1(A4)-D4(A1)] +j4 [D3(A1)-D1(A3)]

For minimum distance 3: 1/2[SL(572)-SR(275)]
[0]
-j2 [D5(A7)-D7(A5)] -j5 [D7(A2)-D2(A7)] -j7 [D2(A5)-D5(A2)]
[2]
+j0 [D5(A7)-D7(A5)] -j5 [D0(A7)+D7(A0)] +j7 [D0(A5)+D5(A0)]
[5]
+j0 [D7(A2)-D2(A7)] +j2 [D0(A7)+D7(A0)] -j7 [D0(A2)+D2(A0)]
[7]
+j0 [D2(A5)-D5(A2)] -j2 [D0(A5)+D5(A0)] +j5 [D0(A2)+D2(A0)]

For minimum distance 4: 1/2[SL(653)+SR(356)]
[1]
-j2 [D4(A7)-D7(A4)] -j4 [D7(A2)-D2(A7)] -j7 [D2(A4)-D4(A2)]
[2]
+j1 [D4(A7)-D7(A4)] -j4 [D1(A7)-D7(A1)] +j7 [D1(A4)-D4(A1)]
[4]
+j1 [D7(A2)-D2(A7)] +j2 [D1(A7)-D7(A1)] -j7 [D1(A2)-D2(A1)]

```
[7]
+j1 [D2(A4)-D4(A2)] -j2 [D1(A4)-D4(A1)] +j4 [D1(A2)-D2(A1)]

For minimum distance  4: 1/2[SL(653)-SR(356)]
[0]
-j3 [D6(A5)-D5(A6)] -j5 [D3(A6)-D6(A3)] -j6 [D5(A3)-D3(A5)]
[3]
+j0 [D6(A5)-D5(A6)] +j5 [D0(A6)+D6(A0)] -j6 [D0(A5)+D5(A0)]
[5]
+j0 [D3(A6)-D6(A3)] -j3 [D0(A6)-D6(A0)] +j6 [D0(A3)+D3(A0)]
[6]
+j0 [D5(A3)-D3(A5)] +j3 [D0(A5)+D5(A0)] -j5 [D0(A3)+D3(A0)]

For minimum distance  5: 1/2[SL(145)+SR(541)]
[2]
-j3 [D7(A6)-D6(A7)] +j6 [D7(A3)-D3(A7)] +j7 [D3(A6)-D6(A3)]
[3]
+j2 [D7(A6)-D6(A7)] -j6 [D7(A2)-D2(A7)] +j7 [D6(A2)-D2(A6)]
[6]
-j2 [D7(A3)-D3(A7)] +j3 [D7(A2)-D2(A7)] +j7 [D2(A3)-D3(A2)]
[7]
-j2 [D3(A6)-D6(A3)] -j3 [D6(A2)-D2(A6)] -j6 [D2(A3)-D3(A2)]

For minimum distance  5: 1/2[SL(145)-SR(541)]
[0]
-j1 [D4(A5)-D5(A4)] -j4 [D5(A1)-D1(A5)] -j5 [D1(A4)-D4(A1)]
[1]
+j0 [D4(A5)-D5(A4)] -j4 [D0(A5)+D5(A0)] +j5 [D0(A4)+D4(A0)]
[4]
+j0 [D5(A1)-D1(A5)] +j1 [D0(A5)+D5(A0)] -j5 [D0(A1)+D1(A0)]
[5]
+j0 [D1(A4)-D4(A1)] -j1 [D0(A4)+D4(A0)] +j4 [D0(A1)+D1(A0)]

For minimum distance  6: 1/2[SL(246)+SR(642)]
[1]
+j3 [D5(A7)-D7(A5)] +j5 [D7(A3)-D3(A7)] -j7 [D5(A3)-D3(A5)]
[3]
-j1 [D5(A7)-D7(A5)] +j5 [D1(A7)-D7(A1)] +j7 [D5(A1)-D1(A5)]
[5]
-j1 [D7(A3)-D3(A7)] -j3 [D1(A7)-D7(A1)] -j7 [D3(A1)-D1(A3)]
[7]
+j1 [D5(A3)-D3(A5)] -j3 [D5(A1)-D1(A5)] +j5 [D3(A1)-D1(A3)]

For minimum distance  6: 1/2[SL(246)-SR(642)]
[0]
-j2 [D4(A6)-D6(A4)] -j4 [D6(A2)-D2(A6)] -j6 [D2(A4)-D4(A2)]
[2]
+j0 [D4(A6)-D6(A4)] -j4 [D0(A6)+D6(A0)] +j6 [D0(A4)+D4(A0)]
[4]
+j0 [D6(A2)-D2(A6)] +j2 [D0(A6)+D6(A0)] -j6 [D0(A2)+D2(A0)]
[6]
+j0 [D2(A4)-D4(A2)] -j2 [D0(A4)+D4(A0)] +j4 [D0(A2)+D2(A0)]

For minimum distance  7: 1/2[SL(347)+SR(743)]
[1]
-j2 [D6(A5)-D5(A6)] +j5 [D6(A2)-D2(A6)] +j6 [D2(A5)-D5(A2)]
[2]
```

+j1 [D6(A5)-D5(A6)] -j5 [D6(A1)-D1(A6)] +j6 [D5(A1)-D1(A5)]
[5]
-j1 [D6(A2)-D2(A6)] +j2 [D6(A1)-D1(A6)] +j6 [D1(A2)-D2(A1)]
[6]
-j1 [D2(A5)-D5(A2)] -j2 [D5(A1)-D1(A5)] -j5 [D1(A2)-D2(A1)]

For minimum distance 7: 1/2[SL(347)-SR(743)]
[0]
-j3 [D4(A7)-D7(A4)] -j4 [D7(A3)-D3(A7)] -j7 [D3(A4)-D4(A3)]
[3]
+j0 [D4(A7)-D7(A4)] -j4 [D0(A7)+D7(A0)] +j7 [D0(A4)+D4(A0)]
[4]
+j0 [D7(A3)-D3(A7)] +j3 [D0(A7)+D7(A0)] -j7 [D0(A3)+D3(A0)]
[7]
+j0 [D3(A4)-D4(A3)] -j3 [D0(A4)+D4(A0)] +j4 [D0(A3)+D3(A0)]

The next turn on the Electrodynamics roadmap is to recast the work-force equation in an integrable form such that every product term contains an outside differentiation. Then we integrate this over the spatial 7-volume. Terms with outside scalar differentiation become time derivatives of volume integrals. Terms with outside spatial differentiations are converted to surface integrals for the surfaces enclosing the 7-volumes. We can equate the also 7-volume integrated original work-force action with this equivalent but different integral form. The result is an expression for the conservation of energy in the scalar portion, and conservation of momentum in the vector portion.

In classical Electrodynamics, this process is expressing in part the work-force as the differential contraction of the stress-energy-momentum tensor. We will require the resultant differentiations to be identical in the Octonion representation to claim victory expressing Electrodynamics within the algebra of Octonions.

Examining terms in the classical Electrodynamics stress-energy-momentum tensor, we see that every term is the product of two field components. This suggests we look to change

Di(Aj)DkDl(Am) (ui uj)(uk (ul um))

to a form

Di [Dj(Ak)Dl(Am)] ui [(uj uk)(ul um)]

where ijklm in the first has no connection to ijklm of the second, they are just indices. Does it look easy? Well it is not when you change the basis unit product history.

Without major clues, this would be a daunting task even with a computer running a symbolic algebra program highly tuned to do the heavy lifting for the Octonion products. Been there, done that. The job becomes much easier when we put our faith in the Law of Octonion Algebraic Invariance.

The full measure of algebraic invariant content in F*j was just what we were looking for to fulfill our expectations for work-force based on classical Electrodynamics. We will be well served by expecting a match from the full compliment of algebraic invariant basis unit products for the product history

ui [(uj uk)(ul um)]

There are five ways to roll algebraic invariants with this basis unit product progression. I will not repeat them here. They are available in a PDF on my website. Instead, I will present something not covered in the PDF, a "C" like pseudo-code procedural outline of how to crank them all out with proper signs to produce an identity with the work-force form already presented.

The pseudo-code has two loops. One cranks out differentiations of forms found off-diagonal in the classic stress-energy-momentum tensor. The second covers differentiations of forms found on-diagonal in the tensor. Of course there is more than this involved in the Octonion formation.

Take Ti as the "i"th basis unit component for the modified Action Function. For the off-diagonal do this

```
For all unequal i,j,k
{
  if j = 0 or (k != 0 and (uj uk) = +/- ui)
    s = -1
  else
    s = +1

  Ti = Ti + s*Dj[ {Dj(Ak)uj uk + Dk(Aj)uk uj}
                   * {Di(Ak)ui uk + Dk(Ai)uk ui} ]
}
```

The on-diagonal forms are expressed by

```
For i=0 to 7
{
  For j=0 to 6
  {
    For k=j+1 to 7
    {
      if i=0
        s = +1
      else
      {
         if j=0
         {
            if k=i
               s = +1
            else
               s = -1
```

```
        }
        else
        {
           if j=i or k=i
              s = -1
           else
              s = +1
        }
     }

     Ti = Ti + s*Di[ {Dj(Ak)uj uk + Dk(Aj)uk uj}
                      *{Dj(Ak)uj uk + Dk(Aj)uk uj} ]
   }
  }
}
```

Since we have algebraic invariants in every product term, it does not matter which algebra is used, all give the same result.

Lets identify some of the terms in the off-diagonal process.

If j=0 we have the time rate of change in the extended Poynting Vector for component ui. If i=0 we have the seven terms in the divergence of the Poynting Vector for unit u0. If k=0, we have the product of two irrotational field components, like the familiar ExEy. If none of ijk are 0, we have the product of two rotational field components, like BxBy.

The on-diagonal terms are all differentiations of products of like field components representing partial energy densities. For i=0 this is simply the negative of the time rate of change in total energy density. For i not zero, it is a bit more than the gradient of energy density as it is for classical Electrodynamics.

One only gets a taste of the eloquence of the integrable form for the work-force equations. I will present them in their full glory in the next installment.

Rick Lockyer

www.octospace.com

For more information see
http://www.octospace.com/files/Octonion_Algebra_and_its_Connection_to_Physics.pdf