Using the differential operator notation developed in the last installment, we can write down the field expressions.

I will use the following algebra prototypes

Right Octonion Algebra

```
(123) (761) (541)
      (572) (642)
      (653) (743)
```

Left Octonion Algebra

```
(123) (761) (145)
      (572) (246)
      (653) (347)
```

Take Octonion A(r) = ui Ai sum i as the potential functions from which the left field form may be made by simple left product

left F(r) = D*A

where * is multiplication defined by the choice of algebra.

I will use [j] where j implies result unit uj is applied to the set of product terms that follow. With this in mind, the left field for Right Octonion prototype algebra as

```
[0]
+D0(A0)-D1(A1)-D2(A2)-D3(A3)-D4(A4)-D5(A5)-D6(A6)-D7(A7)
[1]
+[D0(A1)+D1(A0)]
+[D2(A3)-D3(A2)]+[D5(A4)-D4(A5)]+[D7(A6)-D6(A7)]
[2]
+[D0(A2)+D2(A0)]
+[D3(A1)-D1(A3)]+[D6(A4)-D4(A6)]+[D5(A7)-D7(A5)]
[3]
+[D0(A3)+D3(A0)]
+[D1(A2)-D2(A1)]+[D7(A4)-D4(A7)]+[D6(A5)-D5(A6)]
[4]
+[D0(A4)+D4(A0)]
+[D1(A5)-D5(A1)]+[D2(A6)-D6(A2)]+[D3(A7)-D7(A3)]
[5]
+[D0(A5)+D5(A0)]
+[D7(A2)-D2(A7)]+[D4(A1)-D1(A4)]+[D3(A6)-D6(A3)]
[6]
+[D0(A6)+D6(A0)]
+[D5(A3)-D3(A5)]+[D4(A2)-D2(A4)]+[D1(A7)-D7(A1)]
[7]
+[D0(A7)+D7(A0)]
+[D6(A1)-D1(A6)]+[D4(A3)-D3(A4)]+[D2(A5)-D5(A2)]
```

The scalar [0] term does not seem to have a home in physics. Rather than requiring it to be explicitly zero, I will dodge the issue by removing it algebraically by subtraction of the conjugate of the field followed by division by 2. It would have been nice if the Analogous Lorentz Condition which follows below would have matched the [0] term, but it does not. We could have matched what appears below by defining D as its conjugate, but when this is carried through into the formation of the required form of the Lorentz Condition, it too will change to the identical form of the [0] term above, they both toggle.

Notice the vector field components each have a single irrotational field form and three rotational field forms.

If we closely look at the traditional Electric and Magnetic field types of classical Electrodynamics and their product forms when the xyz system handedness is allowed to change, we find Bi Bj products have the same intrinsic characteristics as the original B components, they will change physical orientation when the handedness of the xyz system changes.

The Ej Ej products do not have the same characteristics as the original E components, the original components do not change physical orientation when the xyz handedness changes but the products do. The products Ei Bj do not change their physical orientation when the xyz handedness changes.

If we are to connect Electrodynamics with Octonion Algebras, we must maintain these intrinsic characteristics of the E and B fields. First things first however. We must first make the extra-algebraic connection between the basis units of the Octonions and our physical xyz space. As shown above, I have chosen the basis unit u4 as the go-between for basis triplets {123} and {567}. The triplet {123} is Octonion permutation (123) yielding a closed set rule for unit product. B field components and their ilk are closed under multiplication, so we must make a connection between the B field and permutation (123). The E field component products are not closed form multiplication, in fact their products match up with the intrinsic characteristics of the B components. {567} is not an Octonion permutation, and any product of two members of {567} is in {123}.

So we have a match if we attach the B field to {123} and the E field to {567}. We want a consistent quad of potential functions for generating both E and B. The single irrotational form in field units u5, u6 and u7 force the selection for B to one of three rotational forms found in each field units u1, u2 and u3. We then have A0, A5, A6 and A7 be the potential functions for the Octonion cover of Electrodynamics and the following field definitions

E field =
+[D0(A5)+D5(A0)] u5
+[D0(A6)+D6(A0)] u6
+[D0(A7)+D7(A0)] u7

B field =

+[D7(A6)-D6(A7)] u1
+[D5(A7)-D7(A5)] u2
+[D6(A5)-D5(A6)] u3

Keep in mind that the Octonion B field form is an algebraic variant, it takes on these signs only for our chosen Right Octonion Algebra and selection of differentiation from the left. The Octonion E field is an algebraic invariant, it always has this form.

The main point of my series of discussion threads is to answer the question "Why Study Octonion Algebras?". The most compelling reason is now before us. The fundamental construction of Octonion Algebras bolts right up to the necessary structure for the electric and magnetic fields of Electrodynamics, even when we look beneath the surface definitions to intrinsic characteristics that are unfortunately often overlooked.

The classical tensorial treatment of the "electromagnetic" field pays homage to the point that the two fields are different enough to require a form that parks their six individual components in separate mathematically distinct locations in the 4^2=16 receptacles of the 4x4 field tensor. I will argue this increase in rank going from vector to second rank tensor is a manifestation of the assumed dimension of the vector space, and not anything more.

If we start with more than four dimensions, we have enough dimensionality to span six distinctly different component types without resorting to rank change.

If all we were looking for is a different and maybe better way to do Electrodynamics, we would have a big "So What" moment at this point. I am not. The task has always been to look at Electrodynamics as a means to an end rather than the end itself. The goal is unification of Electrodynamics and Gravitation. So now I question of the treatment of Gravity in General Relativity?

When Einstein went to curvature of space itself for a mathematical treatment of Gravitation, he did so because there was no other place to put it in the framework of a four dimensional space. The singular method for a central force was already taken by the charge*(electric field) force of Electrodynamics.

I have always had a problem with this. In my humble opinion, the mathematical concept of "space" is a fundamental "first principles" notion that lies beneath the methods of application used to describe what we see and try to make sense out of physical reality. "Space" is to "physical reality" as "marble" is to "a particular sculpture". It is the "foundation" for a house, it is not the "house" itself.

So am I claiming General Relativity is "wrong"? Hardly so. I would think that anyone with enough physics education to be in a position to weigh in on the merits of GR would have come across "generalized coordinates" in their studies. If you did not pick up on the fundamental demonstrated concept that mathematics is robust, and there are normally a number of different mathematical formalisms that can be brought to bear, go back to school and take the mechanics classes again.

I do not claim General Relativity, Quantum Mechanics, Quantum Electrodynamics, The Standard Model, String Theory and the like are "wrong". Each, just like my use of Octonion Algebra, are all different "generalized coordinate systems". All have merit, and all have a following of believers. The parallel to religion is striking. The believers take it on faith that their view of reality is correct, and that all of the non-believers are at best incorrect. At worst and too often the believers portray the non-believers as having something wrong about them.

Until you can prove to a certainty that your religion is the only "true" one, do not look down on those who do not share your beliefs. This goes equally for the study of God and Physics.

But I digress. Getting back to my religion, the Octonion field expressions have additional irrotational forms in the {123} unit triplet we have associated with physical xyz. I am not in a position to demonstrate the "House of Gravity" since I am still working on its "foundation". At this point I have no problem taking it on faith that these are the forms for the Gravitational Field, and moving on from there.

So why do I believe so strongly in Octonions? I took notice of the Octonions when I first realized the fundamental structure of its multiplication rules were just what I needed and was looking for to apply to Electrodynamics, while using a system with dimension greater than four and likely eight. I got religion when I found my musing over the application of the sixteen different forms these structural rules may take was not confronted by physical reality. The fundamental truth and beauty of this notion manifested itself in the "Law of Octonion Algebraic Invariance" which I profess. I became an Octonion Holy Roller when I was able to apply this law successfully to rewrite the Octonion Work-Force Action Equations in an integrable form which allowed discovery of the equations for Octonion Conservation of Energy and Momentum. Lets continue on the progression towards understanding this.

The next move along the Electrodynamics roadmap is coming up with the Octonion 8-current density. Anticipating what will be discovered in the conservation equations and what is present in the decoupling of the classical form of the 4-current density, we look for a form including an extension to what we have in the 4D Lorentz Condition. The ideal form for the 8-current would involve a second application of the Octonion Ensemble Derivative on a form for the Octonion Field Equations. As previously mentioned, we need to dispatch the scalar field component algebraically via conjugate subtraction.

At this point I would like to diverge from the presentation in the PDF available on my website. There, I absorbed the 8-current scale factor -4pi within my description. In hindsight, perhaps I should have only absorbed +4pi and negated the definition I gave for current to be more closely aligned with tradition. I will do this here and continue through the conservation derivation. Please take notice what I present now for current through conservation is the PDF presentations scaled by (-1).

Take the operator E as the entire Octonion ensemble derivative and /E as its conjugate form and try

j = 1/4 { E[E(A) - (/A)/E] + [(A)E - /E(/A)]E }

The scalar field is removed within each of the square brackets. Adding the commutated forms removes unwanted terms from the result. Here, non-commutation of Octonion products is not feared, it is exploited.

If we define the negative of the Octonion extension of the D'Alembertian as the scalar operator form that follows, sticking to the intrinsic variables r hence D operators

-D^2=D0^2-D1^2-D2^2-D3^3-D4^2-D5^2-D6^2-D7^2

and take the Octonion extension to the Lorentz Condition as

LC=D0(A0)+D1(A1)+D2(A2)+D3(A3)+D4(A4)+D5(A5)+D6(A6)+D7(A7)

we will have for j unit [i] after doing the math above

ji ui = -D^2(Ai) ui + Di(LC) ui

LC involves all potential components. Just as with the classical Electrodynamics approach, the 8-current is decoupled into a function of only the like unit potential function component if the Lorentz Condition is zero.

LC is a scalar like unit product and therefore is an Octonion Algebraic Invariant. As such, Di(LC) ui is also an Octonion Algebraic Invariant. Looking at the extended D'Alembertian, it also is a scalar Octonion Algebraic Invariant and therefore the full 8-current is an Octonion Algebraic Invariant.

In the next installment, I will finish up the Octonion Work-Force Action Functions and their morphing to the Octonion Conservation of Energy and Momentum Equations.

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For more information see
http://www.octospace.com/files/Octonion_Algebra_and_its_Connection_to_Physics.pd
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