

Why should anyone bother with Octonion Algebra? After all, it has been dismissed by many intelligent people as being a mathematically interesting algebra, but with little practical application. Some have called it a "lost cause" in physics, others have said they would not recommend it as an area of study for a graduate thesis. Some have claimed a suitable calculus on Octonions cannot be had because the notions of complex regularity do not extend to either the Quaternions or the Octonions. Since Octonions are not an associative algebra for multiplication, algebraic principles near and dear to the faith of many cannot be applied, so they shy away. Many dislike the complexity, where  $A*B$  might be 64 product terms formed by 44 different multiplication rules (42 through permutation rules, scalar basis unit, like basis unit).

How many significant representations of Octonion Algebra are there? Depending on who you ask, the answer is 1 to 480. It somewhat comes down to a what person's definition of "significant" is, and how deep their knowledge is on the subject. The person that answers one has missed something, the person that answers 480 sees extra-algebraic qualities as significant, like aliasing of basis unit names.

If you forego aliasing, there cannot possibly be more than 128 different algebras. If you realize there are fully compliant Octonion representations that are not isomorphic algebras, you will understand the answer cannot be one. If you look closely at the 128 possible non-aliasing representations, you will find not all exhibit as an alternative algebra as Octonion Algebras must. Only sixteen do, and they fall into two non-isomorphic groups of eight.

So what do we have here? No commutation or associativity for multiplication, 16 ways to define 42 of the different multiplication rules, product term counts that increase eightfold on each multiply. How could this much complexity and variability ever have anything to do with nature?

The complexity is only a problem if your weapons of choice are your brains, a pencil and some paper. This stuff will be a prescription for permanent writer's cramp. The tedium of keeping all the basis unit product rules straight may drive one to deep psychosis. From the times of Hamilton and Graves up to recent history, the complexity was a big obstacle for most, and a barrier to understanding for many.

Today however, suitable computing power is readily available. Unfortunately, suitable software is not widely available. I had to write my own symbolic algebra software. There was some advantage to this, since as my knowledge grew, so did my needs. I was able to extend the software to provide additional functionality as needed, rather than having to live with what available software could do for me. It is my sincere hope that the fundamentals I present here and additional concepts I will continue to post on my website will find their way into available symbolic algebra

software. One must immerse themselves deeply in the algebra in order to fully appreciate it. Proper tools are a must.

So it might be possible to deal with the complexity. What about the variability? Once you crack the complexity problem, you begin to see the whole forest rather than the individual trees. The variability can be expressed by simple rules that once understood, provide great insight into the connection between Octonion Algebra and physical reality.

When we attempt to connect a brand of mathematical structure that has variability in its fundamental construction to a singular physical reality, we are left with two choices. We could demonstrate one structure is applicable and justify it over the others, or we can conclude the variability does not matter by demanding the same result for every possible representation choice. There is nothing we can come up with to justify one of the 16 Octonion Algebras as better than another, we are left with the latter choice.

This requirement, which I call "The Law Of Octonion Algebraic Invariance" is a restriction on the form physically significant Octonion mathematical expressions can take. The question is whether or not these restrictions are constricting or enlightening. I hope to convince you they are quite enlightening.

The differential equations of Electrodynamics are well known and understood. It is possible to describe this formalism within the structure of Octonion Algebra. The extremely nice thing about this is that Electrodynamics only takes up a portion of the connection. There is enough present to show how things must be, Electrodynamics provides an excellent roadmap. The differential equations that are not part of Electrodynamics have form that is mandated by the structure of Octonion Algebra as well as the needs of Electrodynamics. Within these additional differential equations is another central force, provided by the added dimensionality of Octonions over 4D space-time. Perhaps this is gravity.

This is reason enough to seriously consider Octonion algebra for a mathematical treatment of physics. I hope to persuade you to embrace the Octonions as I have with the series of discussion threads to follow. They will cover information available on my website with more detail.

Why Octonions 2 will cover the definition of Left and Right Octonion Algebras and their 8 variations. All 8 variations in each type are shown to be algebraic isomorphisms. Left and Right Octonion Algebras are shown to not be algebraically isomorphic.

Why Octonions 3 will cover the consequences of the product rule variances between the 16 forms of Octonion Algebra.

The Law of Octonion Algebraic Invariance is discussed.

Why Octonions 4 will cover the proper definition of Octonion differentiation, the Octonion Ensemble Derivative. An improved form of the chain rule is presented. The diffeomorphic definition for the Ensemble Derivative provides its coordinate transformation properties and demonstrates the Law of Octonion Algebraic Invariance is coordinate system invariant.

Why Octonions 5 will introduce the connection between Octonion Algebra and the intrinsic characteristics of the two fields of Electrodynamics. It will cover the definition for Octonion Fields and 8-current density.

Why Octonions 6 will cover the Octonion Work-Force Action Function and resultant Octonion Conservation of Energy and Momentum.

Why Octonions 7 presents the integrable form of the Octonion Work-Force Action Function in its full glory.

Why Octonions 8 contains concluding remarks

This is fertile ground.

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[http://www.octospace.com/files/Octonion\\_Algebra\\_and\\_its\\_Connection\\_to\\_Physics.pdf](http://www.octospace.com/files/Octonion_Algebra_and_its_Connection_to_Physics.pdf)