

Octonion Torque Variances

In classical mechanics, torque is represented by the 3D vector cross product of the position vector \mathbf{r} with the force \mathbf{f} , that is $\mathbf{r} \times \mathbf{f}$. The cross product of Octonion position \mathbf{r} and Octonion force \mathbf{f} is represented by a subset of product terms from the full up Octonion product $\mathbf{r} * \mathbf{f}$. These product terms are those where the component indices of both \mathbf{r} and \mathbf{f} are unequal and not scalar. In any of the 16 possible Octonion Algebra representations ^{[1][2]}, each of these product types anti-commutes. They may therefore be separated from the other product terms in the full Octonion product with the following expression for torque

$$\mathbf{T} = \frac{1}{2} [\mathbf{r} * \mathbf{f} - \mathbf{f} * \mathbf{r}]$$

Torque is a physical observable, and as such, its Octonion representation must be an Octonion Algebraic Invariant ^{[1][3]}. The expression above for torque will separate via the Octonion Variance Sieve ^{[1][4]} into invariant and variant product term sets. As such, all variant product term sets must be equated to zero independent of whether or not the system has non-zero torque. For a closed and stable physical system, even the invariant product term set for torque must be equated to zero to conserve angular momentum.

The Octonion expression for force is ^{[1][5]}

$$\text{Invariant } \{ \mathbf{F} * \mathbf{j} \} = \mathbf{f}$$

Here \mathbf{F} is the expression for the (vector component only) field components derived from the 8-potential functions as a single application of the Ensemble Derivative ^{[1][6]} on the 8-potentials and \mathbf{j} is the 8-current. The 8-current form which includes the 8 dimensional equivalent of the D'Alembertian and 8-gradient of the Analogous Lorentz Condition, was shown in the references to be a straight up full algebraic invariant. As such, it may be represented in any algebraic variance sieve as simply \mathbf{j} rather than a history of product orderings. The field representation has a mix of variant (like the magnetic field) and invariant (like the electric field). Its full product history must be and was included in the variance sieve of the general force.

Similarly, the proper sieve of the torque must include the full product history of all constituents where octonion multiplications may have resulted in intermediate variant terms. Thus, we must use the full product $\mathbf{F} * \mathbf{j}$ in the torque variance sieve and not just its invariant product terms. The torque expression is thus

$$\mathbf{T} = \frac{1}{2} [\mathbf{r} * [\mathbf{F} * \mathbf{j}] - [\mathbf{F} * \mathbf{j}] * \mathbf{r}]$$

Running this expression through the Octonion Variance Sieve gives the following component results for rectilinear result basis [n]

Invariant Product Terms

[1]

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$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_0(a_1)+D_1(a_0)]$$

$$\begin{aligned} &-j_1r_1 [D_0(a_1)+D_1(a_0)] \\ &-j_1r_2 [D_0(a_2)+D_2(a_0)] \\ &-j_1r_3 [D_0(a_3)+D_3(a_0)] \\ &-j_1r_4 [D_0(a_4)+D_4(a_0)] \\ &-j_1r_5 [D_0(a_5)+D_5(a_0)] \\ &-j_1r_6 [D_0(a_6)+D_6(a_0)] \\ &-j_1r_7 [D_0(a_7)+D_7(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_2 [D_1(a_2)-D_2(a_1)] \\ &+j_0r_3 [D_1(a_3)-D_3(a_1)] \\ &+j_0r_4 [D_1(a_4)-D_4(a_1)] \\ &+j_0r_5 [D_1(a_5)-D_5(a_1)] \\ &+j_0r_6 [D_1(a_6)-D_6(a_1)] \\ &+j_0r_7 [D_1(a_7)-D_7(a_1)] \end{aligned}$$

[2]

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_0(a_2)+D_2(a_0)]$$

$$\begin{aligned} &-j_2r_1 [D_0(a_1)+D_1(a_0)] \\ &-j_2r_2 [D_0(a_2)+D_2(a_0)] \\ &-j_2r_3 [D_0(a_3)+D_3(a_0)] \\ &-j_2r_4 [D_0(a_4)+D_4(a_0)] \\ &-j_2r_5 [D_0(a_5)+D_5(a_0)] \\ &-j_2r_6 [D_0(a_6)+D_6(a_0)] \\ &-j_2r_7 [D_0(a_7)+D_7(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_1 [D_2(a_1)-D_1(a_2)] \\ &+j_0r_3 [D_2(a_3)-D_3(a_2)] \\ &+j_0r_4 [D_2(a_4)-D_4(a_2)] \\ &+j_0r_5 [D_2(a_5)-D_5(a_2)] \\ &+j_0r_6 [D_2(a_6)-D_6(a_2)] \\ &+j_0r_7 [D_2(a_7)-D_7(a_2)] \end{aligned}$$

[3]

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_0(a_3)+D_3(a_0)]$$

$$\begin{aligned} &-j_3r_1 [D_0(a_1)+D_1(a_0)] \\ &-j_3r_2 [D_0(a_2)+D_2(a_0)] \\ &-j_3r_3 [D_0(a_3)+D_3(a_0)] \\ &-j_3r_4 [D_0(a_4)+D_4(a_0)] \\ &-j_3r_5 [D_0(a_5)+D_5(a_0)] \\ &-j_3r_6 [D_0(a_6)+D_6(a_0)] \\ &-j_3r_7 [D_0(a_7)+D_7(a_0)] \end{aligned}$$

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$$\begin{aligned}
 &+j_0r_1 [D_3(a_1)-D_1(a_3)] \\
 &+j_0r_2 [D_3(a_2)-D_2(a_3)] \\
 &+j_0r_4 [D_3(a_4)-D_4(a_3)] \\
 &+j_0r_5 [D_3(a_5)-D_5(a_3)] \\
 &+j_0r_6 [D_3(a_6)-D_6(a_3)] \\
 &+j_0r_7 [D_3(a_7)-D_7(a_3)]
 \end{aligned}$$

[4]

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_0(a_4)+D_4(a_0)]$$

$$\begin{aligned}
 &-j_4r_1 [D_0(a_1)+D_1(a_0)] \\
 &-j_4r_2 [D_0(a_2)+D_2(a_0)] \\
 &-j_4r_3 [D_0(a_3)+D_3(a_0)] \\
 &-j_4r_4 [D_0(a_4)+D_4(a_0)] \\
 &-j_4r_5 [D_0(a_5)+D_5(a_0)] \\
 &-j_4r_6 [D_0(a_6)+D_6(a_0)] \\
 &-j_4r_7 [D_0(a_7)+D_7(a_0)]
 \end{aligned}$$

$$\begin{aligned}
 &+j_0r_1 [D_4(a_1)-D_1(a_4)] \\
 &+j_0r_2 [D_4(a_2)-D_2(a_4)] \\
 &+j_0r_3 [D_4(a_3)-D_3(a_4)] \\
 &+j_0r_5 [D_4(a_5)-D_5(a_4)] \\
 &+j_0r_6 [D_4(a_6)-D_6(a_4)] \\
 &+j_0r_7 [D_4(a_7)-D_7(a_4)]
 \end{aligned}$$

[5]

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_0(a_5)+D_5(a_0)]$$

$$\begin{aligned}
 &-j_5r_1 [D_0(a_1)+D_1(a_0)] \\
 &-j_5r_2 [D_0(a_2)+D_2(a_0)] \\
 &-j_5r_3 [D_0(a_3)+D_3(a_0)] \\
 &-j_5r_4 [D_0(a_4)+D_4(a_0)] \\
 &-j_5r_5 [D_0(a_5)+D_5(a_0)] \\
 &-j_5r_6 [D_0(a_6)+D_6(a_0)] \\
 &-j_5r_7 [D_0(a_7)+D_7(a_0)]
 \end{aligned}$$

$$\begin{aligned}
 &+j_0r_1 [D_5(a_1)-D_1(a_5)] \\
 &+j_0r_2 [D_5(a_2)-D_2(a_5)] \\
 &+j_0r_3 [D_5(a_3)-D_3(a_5)] \\
 &+j_0r_4 [D_5(a_4)-D_4(a_5)] \\
 &+j_0r_6 [D_5(a_6)-D_6(a_5)] \\
 &+j_0r_7 [D_5(a_7)-D_7(a_5)]
 \end{aligned}$$

[6]

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_0(a_6)+D_6(a_0)]$$

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$$\begin{aligned}
 & -j_6 r_1 [D_0(a_1)+D_1(a_0)] \\
 & -j_6 r_2 [D_0(a_2)+D_2(a_0)] \\
 & -j_6 r_3 [D_0(a_3)+D_3(a_0)] \\
 & -j_6 r_4 [D_0(a_4)+D_4(a_0)] \\
 & -j_6 r_5 [D_0(a_5)+D_5(a_0)] \\
 & -j_6 r_6 [D_0(a_6)+D_6(a_0)] \\
 & -j_6 r_7 [D_0(a_7)+D_7(a_0)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_0 r_1 [D_6(a_1)-D_1(a_6)] \\
 & +j_0 r_2 [D_6(a_2)-D_2(a_6)] \\
 & +j_0 r_3 [D_6(a_3)-D_3(a_6)] \\
 & +j_0 r_4 [D_6(a_4)-D_4(a_6)] \\
 & +j_0 r_5 [D_6(a_5)-D_5(a_6)] \\
 & +j_0 r_7 [D_6(a_7)-D_7(a_6)]
 \end{aligned}$$

[7]

$$[+j_1 r_1 + j_2 r_2 + j_3 r_3 + j_4 r_4 + j_5 r_5 + j_6 r_6 + j_7 r_7] [D_0(a_7)+D_7(a_0)]$$

$$\begin{aligned}
 & -j_7 r_1 [D_0(a_1)+D_1(a_0)] \\
 & -j_7 r_2 [D_0(a_2)+D_2(a_0)] \\
 & -j_7 r_3 [D_0(a_3)+D_3(a_0)] \\
 & -j_7 r_4 [D_0(a_4)+D_4(a_0)] \\
 & -j_7 r_5 [D_0(a_5)+D_5(a_0)] \\
 & -j_7 r_6 [D_0(a_6)+D_6(a_0)] \\
 & -j_7 r_7 [D_0(a_7)+D_7(a_0)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_0 r_1 [D_7(a_1)-D_1(a_7)] \\
 & +j_0 r_2 [D_7(a_2)-D_2(a_7)] \\
 & +j_0 r_3 [D_7(a_3)-D_3(a_7)] \\
 & +j_0 r_4 [D_7(a_4)-D_4(a_7)] \\
 & +j_0 r_5 [D_7(a_5)-D_5(a_7)] \\
 & +j_0 r_6 [D_7(a_6)-D_6(a_7)]
 \end{aligned}$$

Variant $\frac{1}{2}$ (SL{123} + SR{123})

[4]

$$\begin{aligned}
 & [+j_5 r_7 - j_7 r_5] [D_0(a_6)+D_6(a_0)] \\
 & [+j_6 r_5 - j_5 r_6] [D_0(a_7)+D_7(a_0)] \\
 & [+j_7 r_6 - j_6 r_7] [D_0(a_5)+D_5(a_0)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_0 r_5 [D_7(a_6)-D_6(a_7)] \\
 & +j_0 r_6 [D_5(a_7)-D_7(a_5)] \\
 & +j_0 r_7 [D_6(a_5)-D_5(a_6)]
 \end{aligned}$$

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[5]

$$[+j_4r_6-j_6r_4] [D_0(a_7)+D_7(a_0)]$$

$$[+j_6r_7-j_7r_6] [D_0(a_4)+D_4(a_0)]$$

$$[+j_7r_4-j_4r_7] [D_0(a_6)+D_6(a_0)]$$

$$+j_0r_4 [D_6(a_7)-D_7(a_6)]$$

$$+j_0r_6 [D_7(a_4)-D_4(a_7)]$$

$$+j_0r_7 [D_4(a_6)-D_6(a_4)]$$

[6]

$$[+j_4r_7-j_7r_4] [D_0(a_5)+D_5(a_0)]$$

$$[+j_5r_4-j_4r_5] [D_0(a_7)+D_7(a_0)]$$

$$[+j_7r_5-j_5r_7] [D_0(a_4)+D_4(a_0)]$$

$$+j_0r_4 [D_7(a_5)-D_5(a_7)]$$

$$+j_0r_5 [D_4(a_7)-D_7(a_4)]$$

$$+j_0r_7 [D_5(a_4)-D_4(a_5)]$$

[7]

$$[+j_4r_5-j_5r_4] [D_0(a_6)+D_6(a_0)]$$

$$[+j_5r_6-j_6r_5] [D_0(a_4)+D_4(a_0)]$$

$$[+j_6r_4-j_4r_6] [D_0(a_5)+D_5(a_0)]$$

$$+j_0r_4 [D_5(a_6)-D_6(a_5)]$$

$$+j_0r_5 [D_6(a_4)-D_4(a_6)]$$

$$+j_0r_6 [D_4(a_5)-D_5(a_4)]$$

Variant $\frac{1}{2}$ (SL{123} – SR{123})

[1]

$$+j_0r_2 [D_0(a_3)+D_3(a_0)]$$

$$-j_0r_3 [D_0(a_2)+D_2(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_2(a_3)-D_3(a_2)]$$

$$[+j_2r_4-j_4r_2] [D_3(a_4)-D_4(a_3)]$$

$$[+j_2r_5-j_5r_2] [D_3(a_5)-D_5(a_3)]$$

$$[+j_2r_6-j_6r_2] [D_3(a_6)-D_6(a_3)]$$

$$[+j_2r_7-j_7r_2] [D_3(a_7)-D_7(a_3)]$$

$$[+j_3r_4-j_4r_3] [D_4(a_2)-D_2(a_4)]$$

$$[+j_3r_5-j_5r_3] [D_5(a_2)-D_2(a_5)]$$

$$[+j_3r_6-j_6r_3] [D_6(a_2)-D_2(a_6)]$$

$$[+j_3r_7-j_7r_3] [D_7(a_2)-D_2(a_7)]$$

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$$\begin{aligned}
 &+j_1r_1 [D_3(a_2)-D_2(a_3)] \\
 &+j_1r_2 [D_1(a_3)-D_3(a_1)] \\
 &+j_1r_3 [D_2(a_1)-D_1(a_2)]
 \end{aligned}$$

[2]

$$\begin{aligned}
 &+j_0r_3 [D_0(a_1)+D_1(a_0)] \\
 &-j_0r_1 [D_0(a_3)+D_3(a_0)]
 \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_3(a_1)-D_1(a_3)]$$

$$\begin{aligned}
 &[+j_3r_4-j_4r_3] [D_1(a_4)-D_4(a_1)] \\
 &[+j_3r_5-j_5r_3] [D_1(a_5)-D_5(a_1)] \\
 &[+j_3r_6-j_6r_3] [D_1(a_6)-D_6(a_1)] \\
 &[+j_3r_7-j_7r_3] [D_1(a_7)-D_7(a_1)] \\
 &[+j_1r_4-j_4r_1] [D_4(a_3)-D_3(a_4)] \\
 &[+j_1r_5-j_5r_1] [D_5(a_3)-D_3(a_5)] \\
 &[+j_1r_6-j_6r_1] [D_6(a_3)-D_3(a_6)] \\
 &[+j_1r_7-j_7r_1] [D_7(a_3)-D_3(a_7)]
 \end{aligned}$$

$$\begin{aligned}
 &+j_2r_1 [D_3(a_2)-D_2(a_3)] \\
 &+j_2r_2 [D_1(a_3)-D_3(a_1)] \\
 &+j_2r_3 [D_2(a_1)-D_1(a_2)]
 \end{aligned}$$

[3]

$$\begin{aligned}
 &+j_0r_1 [D_0(a_2)+D_2(a_0)] \\
 &-j_0r_2 [D_0(a_1)+D_1(a_0)]
 \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_1(a_2)-D_2(a_1)]$$

$$\begin{aligned}
 &[+j_1r_4-j_4r_1] [D_2(a_4)-D_4(a_2)] \\
 &[+j_1r_5-j_5r_1] [D_2(a_5)-D_5(a_2)] \\
 &[+j_1r_6-j_6r_1] [D_2(a_6)-D_6(a_2)] \\
 &[+j_1r_7-j_7r_1] [D_2(a_7)-D_7(a_2)] \\
 &[+j_2r_4-j_4r_2] [D_4(a_1)-D_1(a_4)] \\
 &[+j_2r_5-j_5r_2] [D_5(a_1)-D_1(a_5)] \\
 &[+j_2r_6-j_6r_2] [D_6(a_1)-D_1(a_6)] \\
 &[+j_2r_7-j_7r_2] [D_7(a_1)-D_1(a_7)]
 \end{aligned}$$

$$\begin{aligned}
 &+j_3r_1 [D_3(a_2)-D_2(a_3)] \\
 &+j_3r_2 [D_1(a_3)-D_3(a_1)] \\
 &+j_3r_3 [D_2(a_1)-D_1(a_2)]
 \end{aligned}$$

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[4]

$$[+j_1r_2-j_2r_1] [D_3(a_4)-D_4(a_3)]$$

$$[+j_2r_3-j_3r_2] [D_1(a_4)-D_4(a_1)]$$

$$[+j_3r_1-j_1r_3] [D_2(a_4)-D_4(a_2)]$$

$$+j_4r_1 [D_3(a_2)-D_2(a_3)]$$

$$+j_4r_2 [D_1(a_3)-D_3(a_1)]$$

$$+j_4r_3 [D_2(a_1)-D_1(a_2)]$$

[5]

$$[+j_1r_2-j_2r_1] [D_3(a_5)-D_5(a_3)]$$

$$[+j_2r_3-j_3r_1] [D_1(a_5)-D_5(a_1)]$$

$$[+j_3r_1-j_1r_3] [D_2(a_5)-D_5(a_2)]$$

$$+j_5r_1 [D_3(a_2)-D_2(a_3)]$$

$$+j_5r_2 [D_1(a_3)-D_3(a_1)]$$

$$+j_5r_3 [D_2(a_1)-D_1(a_2)]$$

[6]

$$[+j_1r_2-j_2r_1] [D_3(a_6)-D_6(a_3)]$$

$$[+j_2r_3-j_3r_2] [D_1(a_6)-D_6(a_1)]$$

$$[+j_3r_1-j_1r_3] [D_2(a_6)-D_6(a_2)]$$

$$+j_6r_1 [D_3(a_2)-D_2(a_3)]$$

$$+j_6r_2 [D_1(a_3)-D_3(a_1)]$$

$$+j_6r_3 [D_2(a_1)-D_1(a_2)]$$

[7]

$$[+j_1r_2-j_2r_1] [D_3(a_7)-D_7(a_3)]$$

$$[+j_2r_3-j_3r_2] [D_1(a_7)-D_7(a_1)]$$

$$[+j_3r_1-j_1r_3] [D_2(a_7)-D_7(a_2)]$$

$$+j_7r_1 [D_3(a_2)-D_2(a_3)]$$

$$+j_7r_2 [D_1(a_3)-D_3(a_1)]$$

$$+j_7r_3 [D_2(a_1)-D_1(a_2)]$$

Variant $\frac{1}{2}$ (SL{761} + SR{761})

[2]

$$[+j_3r_4-j_4r_3] [D_0(a_5)+D_5(a_0)]$$

$$[+j_4r_5-j_5r_4] [D_0(a_3)+D_3(a_0)]$$

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$$[+j_5r_3-j_3r_5] [D_0(a_4)+D_4(a_0)]$$

$$+j_0r_3 [D_4(a_5)-D_5(a_4)]$$

$$+j_0r_4 [D_5(a_3)-D_3(a_5)]$$

$$+j_0r_5 [D_3(a_4)-D_4(a_3)]$$

[3]

$$[+j_2r_5-j_5r_2] [D_0(a_4)+D_4(a_0)]$$

$$[+j_4r_2-j_2r_4] [D_0(a_5)+D_5(a_0)]$$

$$[+j_5r_4-j_4r_5] [D_0(a_2)+D_2(a_0)]$$

$$+j_0r_2 [D_5(a_4)-D_4(a_5)]$$

$$+j_0r_4 [D_2(a_5)-D_5(a_2)]$$

$$+j_0r_5 [D_4(a_2)-D_2(a_4)]$$

[4]

$$[+j_2r_3-j_3r_2] [D_0(a_5)+D_5(a_0)]$$

$$[+j_3r_5-j_5r_3] [D_0(a_2)+D_2(a_0)]$$

$$[+j_5r_2-j_2r_5] [D_0(a_3)+D_3(a_0)]$$

$$+j_0r_2 [D_3(a_5)-D_5(a_3)]$$

$$+j_0r_3 [D_5(a_2)-D_2(a_5)]$$

$$+j_0r_5 [D_2(a_3)-D_3(a_2)]$$

[5]

$$[+j_2r_4-j_4r_2] [D_0(a_3)+D_3(a_0)]$$

$$[+j_3r_2-j_2r_3] [D_0(a_4)+D_4(a_0)]$$

$$[+j_4r_3-j_3r_4] [D_0(a_2)+D_2(a_0)]$$

$$+j_0r_2 [D_4(a_3)-D_3(a_4)]$$

$$+j_0r_3 [D_2(a_4)-D_4(a_2)]$$

$$+j_0r_4 [D_3(a_2)-D_2(a_3)]$$

Variant $1/2$ (SL{761} – SR{761})

[1]

$$+j_0r_7 [D_0(a_6)+D_6(a_0)]$$

$$-j_0r_6 [D_0(a_7)+D_7(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_7(a_6)-D_6(a_7)]$$

$$[+j_7r_2-j_2r_7] [D_6(a_2)-D_2(a_6)]$$

$$[+j_7r_3-j_3r_7] [D_6(a_3)-D_3(a_6)]$$

$$[+j_7r_4-j_4r_7] [D_6(a_4)-D_4(a_6)]$$

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$$\begin{aligned} & [+j_7r_5-j_5r_7] [D_6(a_5)-D_5(a_6)] \\ & [+j_6r_2-j_2r_6] [D_2(a_7)-D_7(a_2)] \\ & [+j_6r_3-j_3r_6] [D_3(a_7)-D_7(a_3)] \\ & [+j_6r_4-j_4r_6] [D_4(a_7)-D_7(a_4)] \\ & [+j_6r_5-j_5r_6] [D_5(a_7)-D_7(a_5)] \end{aligned}$$

$$\begin{aligned} & +j_1r_1 [D_6(a_7)-D_7(a_6)] \\ & +j_1r_6 [D_7(a_1)-D_1(a_7)] \\ & +j_1r_7 [D_1(a_6)-D_6(a_1)] \end{aligned}$$

[2]

$$\begin{aligned} & [+j_1r_7-j_7r_1] [D_6(a_2)-D_2(a_6)] \\ & [+j_6r_1-j_1r_6] [D_7(a_2)-D_2(a_7)] \\ & [+j_7r_6-j_6r_7] [D_1(a_2)-D_2(a_1)] \end{aligned}$$

$$\begin{aligned} & +j_2r_1 [D_6(a_7)-D_7(a_6)] \\ & +j_2r_6 [D_7(a_1)-D_1(a_7)] \\ & +j_2r_7 [D_1(a_6)-D_6(a_1)] \end{aligned}$$

[3]

$$\begin{aligned} & [+j_1r_7-j_7r_1] [D_6(a_3)-D_3(a_6)] \\ & [+j_6r_1-j_1r_6] [D_7(a_3)-D_3(a_7)] \\ & [+j_7r_6-j_6r_7] [D_1(a_3)-D_3(a_1)] \end{aligned}$$

$$\begin{aligned} & +j_3r_1 [D_6(a_7)-D_7(a_6)] \\ & +j_3r_6 [D_7(a_1)-D_1(a_7)] \\ & +j_3r_7 [D_1(a_6)-D_6(a_1)] \end{aligned}$$

[4]

$$\begin{aligned} & [+j_1r_7-j_7r_1] [D_6(a_4)-D_4(a_6)] \\ & [+j_6r_1-j_1r_6] [D_7(a_4)-D_4(a_7)] \\ & [+j_7r_6-j_6r_7] [D_1(a_4)-D_4(a_1)] \end{aligned}$$

$$\begin{aligned} & +j_4r_1 [D_6(a_7)-D_7(a_6)] \\ & +j_4r_6 [D_7(a_1)-D_1(a_7)] \\ & +j_4r_7 [D_1(a_6)-D_6(a_1)] \end{aligned}$$

[5]

$$\begin{aligned} & [+j_1r_7-j_7r_1] [D_6(a_5)-D_5(a_6)] \\ & [+j_6r_1-j_1r_6] [D_7(a_5)-D_5(a_7)] \\ & [+j_7r_6-j_6r_7] [D_1(a_5)-D_5(a_1)] \end{aligned}$$

$$\begin{aligned} & +j_5r_1 [D_6(a_7)-D_7(a_6)] \\ & +j_5r_6 [D_7(a_1)-D_1(a_7)] \\ & +j_5r_7 [D_1(a_6)-D_6(a_1)] \end{aligned}$$

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[6]

$$+j_0r_1 [D_0(a_7)+D_7(a_0)]$$

$$-j_0r_7 [D_0(a_1)+D_1(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_1(a_7)-D_7(a_1)]$$

$$[+j_1r_2-j_2r_1] [D_7(a_2)-D_2(a_7)]$$

$$[+j_1r_3-j_3r_1] [D_7(a_3)-D_3(a_7)]$$

$$[+j_1r_4-j_4r_1] [D_7(a_4)-D_4(a_7)]$$

$$[+j_1r_5-j_5r_1] [D_7(a_5)-D_5(a_7)]$$

$$[+j_7r_2-j_2r_7] [D_2(a_1)-D_1(a_2)]$$

$$[+j_7r_3-j_3r_7] [D_3(a_1)-D_1(a_3)]$$

$$[+j_7r_4-j_4r_7] [D_4(a_1)-D_1(a_4)]$$

$$[+j_7r_5-j_5r_7] [D_5(a_1)-D_1(a_5)]$$

$$+j_6r_1 [D_6(a_7)-D_7(a_6)]$$

$$+j_6r_6 [D_7(a_1)-D_1(a_7)]$$

$$+j_6r_7 [D_1(a_6)-D_6(a_1)]$$

[7]

$$+j_0r_6 [D_0(a_1)+D_1(a_0)]$$

$$-j_0r_1 [D_0(a_6)+D_6(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_6(a_1)-D_1(a_6)]$$

$$[+j_6r_2-j_2r_6] [D_1(a_2)-D_2(a_1)]$$

$$[+j_6r_3-j_3r_6] [D_1(a_3)-D_3(a_1)]$$

$$[+j_6r_4-j_4r_6] [D_1(a_4)-D_4(a_1)]$$

$$[+j_6r_5-j_5r_6] [D_1(a_5)-D_5(a_1)]$$

$$[+j_1r_2-j_2r_1] [D_2(a_6)-D_6(a_2)]$$

$$[+j_1r_3-j_3r_1] [D_3(a_6)-D_6(a_3)]$$

$$[+j_1r_4-j_4r_1] [D_4(a_6)-D_6(a_4)]$$

$$[+j_1r_5-j_5r_1] [D_5(a_6)-D_6(a_5)]$$

$$+j_7r_1 [D_6(a_7)-D_7(a_6)]$$

$$+j_7r_6 [D_7(a_1)-D_1(a_7)]$$

$$+j_7r_7 [D_1(a_6)-D_6(a_1)]$$

Variant $\frac{1}{2}$ (SL{572} + SR{572})

[1]

$$[+j_3r_6-j_6r_3] [D_0(a_4)+D_4(a_0)]$$

$$[+j_4r_3-j_3r_4] [D_0(a_6)+D_6(a_0)]$$

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$$[+j_6r_4-j_4r_6] [D_0(a_3)+D_3(a_0)]$$

$$+j_0r_3 [D_6(a_4)-D_4(a_6)]$$

$$+j_0r_4 [D_3(a_6)-D_6(a_3)]$$

$$+j_0r_6 [D_4(a_3)-D_3(a_4)]$$

[3]

$$[+j_1r_4-j_4r_1] [D_0(a_6)+D_6(a_0)]$$

$$[+j_4r_6-j_6r_4] [D_0(a_1)+D_1(a_0)]$$

$$[+j_6r_1-j_1r_6] [D_0(a_4)+D_4(a_0)]$$

$$+j_0r_1 [D_4(a_6)-D_6(a_4)]$$

$$+j_0r_4 [D_6(a_1)-D_1(a_6)]$$

$$+j_0r_6 [D_1(a_4)-D_4(a_1)]$$

[4]

$$[+j_1r_6-j_6r_1] [D_0(a_3)+D_3(a_0)]$$

$$[+j_3r_1-j_1r_3] [D_0(a_6)+D_6(a_0)]$$

$$[+j_6r_3-j_3r_6] [D_0(a_1)+D_1(a_0)]$$

$$+j_0r_1 [D_6(a_3)-D_3(a_6)]$$

$$+j_0r_3 [D_1(a_6)-D_6(a_1)]$$

$$+j_0r_6 [D_3(a_1)-D_1(a_3)]$$

[6]

$$[+j_1r_3-j_3r_1] [D_0(a_4)+D_4(a_0)]$$

$$[+j_3r_4-j_4r_3] [D_0(a_1)+D_1(a_0)]$$

$$[+j_4r_1-j_1r_4] [D_0(a_3)+D_3(a_0)]$$

$$+j_0r_1 [D_3(a_4)-D_4(a_3)]$$

$$+j_0r_3 [D_4(a_1)-D_1(a_4)]$$

$$+j_0r_4 [D_1(a_3)-D_3(a_1)]$$

Variant $\frac{1}{2}$ (SL{572} – SR{572})

[1]

$$[+j_2r_5-j_5r_2] [D_7(a_1)-D_1(a_7)]$$

$$[+j_5r_7-j_7r_5] [D_2(a_1)-D_1(a_2)]$$

$$[+j_7r_2-j_2r_7] [D_5(a_1)-D_1(a_5)]$$

$$+j_1r_2 [D_7(a_5)-D_5(a_7)]$$

$$+j_1r_5 [D_2(a_7)-D_7(a_2)]$$

$$+j_1r_7 [D_5(a_2)-D_2(a_5)]$$

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[2]

$$+j_0r_5 [D_0(a_7)+D_7(a_0)]$$

$$-j_0r_7 [D_0(a_5)+D_5(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_5(a_7)-D_7(a_5)]$$

$$[+j_5r_1-j_1r_5] [D_7(a_1)-D_1(a_7)]$$

$$[+j_5r_3-j_3r_5] [D_7(a_3)-D_3(a_7)]$$

$$[+j_5r_4-j_4r_5] [D_7(a_4)-D_4(a_7)]$$

$$[+j_5r_6-j_6r_5] [D_7(a_6)-D_6(a_7)]$$

$$[+j_7r_1-j_1r_7] [D_1(a_5)-D_5(a_1)]$$

$$[+j_7r_3-j_3r_7] [D_3(a_5)-D_5(a_3)]$$

$$[+j_7r_4-j_4r_7] [D_4(a_5)-D_5(a_4)]$$

$$[+j_7r_6-j_6r_7] [D_6(a_5)-D_5(a_6)]$$

$$+j_2r_2 [D_7(a_5)-D_5(a_7)]$$

$$+j_2r_5 [D_2(a_7)-D_7(a_2)]$$

$$+j_2r_7 [D_5(a_2)-D_2(a_5)]$$

[3]

$$[+j_2r_5-j_5r_2] [D_7(a_3)-D_3(a_7)]$$

$$[+j_5r_7-j_7r_5] [D_2(a_3)-D_3(a_2)]$$

$$[+j_7r_2-j_2r_7] [D_5(a_3)-D_3(a_5)]$$

$$+j_3r_2 [D_7(a_5)-D_5(a_7)]$$

$$+j_3r_5 [D_2(a_7)-D_7(a_2)]$$

$$+j_3r_7 [D_5(a_2)-D_2(a_5)]$$

[4]

$$[+j_2r_5-j_5r_2] [D_7(a_4)-D_4(a_7)]$$

$$[+j_5r_7-j_7r_5] [D_2(a_4)-D_4(a_2)]$$

$$[+j_7r_2-j_2r_7] [D_5(a_4)-D_4(a_5)]$$

$$+j_4r_2 [D_7(a_5)-D_5(a_7)]$$

$$+j_4r_5 [D_2(a_7)-D_7(a_2)]$$

$$+j_4r_7 [D_5(a_2)-D_2(a_5)]$$

[5]

$$+j_0r_7 [D_0(a_2)+D_2(a_0)]$$

$$-j_0r_2 [D_0(a_7)+D_7(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_7(a_2)-D_2(a_7)]$$

$$[+j_7r_1-j_1r_7] [D_2(a_1)-D_1(a_2)]$$

$$[+j_7r_3-j_3r_7] [D_2(a_3)-D_3(a_2)]$$

$$[+j_7r_4-j_4r_7] [D_2(a_4)-D_4(a_2)]$$

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$$\begin{aligned}
 & [+j_7r_6-j_6r_7] [D_2(a_6)-D_6(a_2)] \\
 & [+j_2r_1-j_1r_2] [D_1(a_7)-D_7(a_1)] \\
 & [+j_2r_3-j_3r_2] [D_3(a_7)-D_7(a_3)] \\
 & [+j_2r_4-j_4r_2] [D_4(a_7)-D_7(a_4)] \\
 & [+j_2r_6-j_6r_2] [D_6(a_7)-D_7(a_6)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_5r_2 [D_7(a_5)-D_5(a_7)] \\
 & +j_5r_5 [D_2(a_7)-D_7(a_2)] \\
 & +j_5r_7 [D_5(a_2)-D_2(a_5)]
 \end{aligned}$$

[6]

$$\begin{aligned}
 & [+j_2r_5-j_5r_2] [D_7(a_6)-D_6(a_7)] \\
 & [+j_5r_7-j_7r_5] [D_2(a_6)-D_6(a_2)] \\
 & [+j_7r_2-j_2r_7] [D_5(a_6)-D_6(a_5)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_6r_2 [D_7(a_5)-D_5(a_7)] \\
 & +j_6r_5 [D_2(a_7)-D_7(a_2)] \\
 & +j_6r_7 [D_5(a_2)-D_2(a_5)]
 \end{aligned}$$

[7]

$$\begin{aligned}
 & +j_0r_2 [D_0(a_5)+D_5(a_0)] \\
 & -j_0r_5 [D_0(a_2)+D_2(a_0)]
 \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_2(a_5)-D_5(a_2)]$$

$$\begin{aligned}
 & [+j_2r_1-j_1r_2] [D_5(a_1)-D_1(a_5)] \\
 & [+j_2r_3-j_3r_2] [D_5(a_3)-D_3(a_5)] \\
 & [+j_2r_4-j_4r_2] [D_5(a_4)-D_4(a_5)] \\
 & [+j_2r_6-j_6r_2] [D_5(a_6)-D_6(a_5)] \\
 & [+j_5r_1-j_1r_5] [D_1(a_2)-D_2(a_1)] \\
 & [+j_5r_3-j_3r_5] [D_3(a_2)-D_2(a_3)] \\
 & [+j_5r_4-j_4r_5] [D_4(a_2)-D_2(a_4)] \\
 & [+j_6r_5-j_5r_6] [D_2(a_6)-D_6(a_2)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_7r_2 [D_7(a_5)-D_5(a_7)] \\
 & +j_7r_5 [D_2(a_7)-D_7(a_2)] \\
 & +j_7r_7 [D_5(a_2)-D_2(a_5)]
 \end{aligned}$$

Variant $\frac{1}{2}$ (SL{653} + SR{653})

[1]

$$\begin{aligned}
 & [+j_2r_4-j_4r_2] [D_0(a_7)+D_7(a_0)] \\
 & [+j_4r_7-j_7r_4] [D_0(a_2)+D_2(a_0)] \\
 & [+j_7r_2-j_2r_7] [D_0(a_4)+D_4(a_0)]
 \end{aligned}$$

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$$\begin{aligned} &+j_0r_2 [D_4(a_7)-D_7(a_4)] \\ &+j_0r_4 [D_7(a_2)-D_2(a_7)] \\ &+j_0r_7 [D_2(a_4)-D_4(a_2)] \end{aligned}$$

[2]

$$\begin{aligned} &[+j_1r_7-j_7r_1] [D_0(a_4)+D_4(a_0)] \\ &[+j_4r_1-j_1r_4] [D_0(a_7)+D_7(a_0)] \\ &[+j_7r_4-j_4r_7] [D_0(a_1)+D_1(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_1 [D_7(a_4)-D_4(a_7)] \\ &+j_0r_4 [D_1(a_7)-D_7(a_1)] \\ &+j_0r_7 [D_4(a_1)-D_1(a_4)] \end{aligned}$$

[4]

$$\begin{aligned} &[+j_1r_2-j_2r_1] [D_0(a_7)+D_7(a_0)] \\ &[+j_2r_7-j_7r_2] [D_0(a_1)+D_1(a_0)] \\ &[+j_7r_1-j_1r_7] [D_0(a_2)+D_2(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_1 [D_2(a_7)-D_7(a_2)] \\ &+j_0r_2 [D_7(a_1)-D_1(a_7)] \\ &+j_0r_7 [D_1(a_2)-D_2(a_1)] \end{aligned}$$

[7]

$$\begin{aligned} &[+j_1r_4-j_4r_1] [D_0(a_2)+D_2(a_0)] \\ &[+j_2r_1-j_1r_2] [D_0(a_4)+D_4(a_0)] \\ &[+j_4r_2-j_2r_4] [D_0(a_1)+D_1(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_1 [D_4(a_2)-D_2(a_4)] \\ &+j_0r_2 [D_1(a_4)-D_4(a_1)] \\ &+j_0r_4 [D_2(a_1)-D_1(a_2)] \end{aligned}$$

Variant $\frac{1}{2}$ (SL{653} – SR{653})

[1]

$$\begin{aligned} &[+j_3r_6-j_6r_3] [D_5(a_1)-D_1(a_5)] \\ &[+j_5r_3-j_3r_5] [D_6(a_1)-D_1(a_6)] \\ &[+j_6r_5-j_5r_6] [D_3(a_1)-D_1(a_3)] \end{aligned}$$

$$\begin{aligned} &+j_1r_3 [D_5(a_6)-D_6(a_5)] \\ &+j_1r_5 [D_6(a_3)-D_3(a_6)] \\ &+j_1r_6 [D_3(a_5)-D_5(a_3)] \end{aligned}$$

[2]

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$$\begin{aligned} & [+j_3r_6-j_6r_3] [D_5(a_2)-D_2(a_5)] \\ & [+j_5r_3-j_3r_5] [D_6(a_2)-D_2(a_6)] \\ & [+j_6r_5-j_5r_6] [D_3(a_2)-D_2(a_3)] \end{aligned}$$

$$\begin{aligned} & +j_2r_3 [D_5(a_6)-D_6(a_5)] \\ & +j_2r_5 [D_6(a_3)-D_3(a_6)] \\ & +j_2r_6 [D_3(a_5)-D_5(a_3)] \end{aligned}$$

$$\begin{aligned} & [3] \\ & +j_0r_6 [D_0(a_5)+D_5(a_0)] \\ & -j_0r_5 [D_6(a_0)+D_0(a_6)] \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_6(a_5)-D_5(a_6)]$$

$$\begin{aligned} & [+j_6r_1-j_1r_6] [D_5(a_1)-D_1(a_5)] \\ & [+j_6r_2-j_2r_6] [D_5(a_2)-D_2(a_5)] \\ & [+j_6r_4-j_4r_6] [D_5(a_4)-D_4(a_5)] \\ & [+j_6r_7-j_7r_6] [D_5(a_7)-D_7(a_5)] \\ & [+j_5r_1-j_1r_5] [D_1(a_6)-D_6(a_1)] \\ & [+j_5r_2-j_2r_5] [D_2(a_6)-D_6(a_2)] \\ & [+j_5r_4-j_4r_5] [D_4(a_6)-D_6(a_4)] \\ & [+j_5r_7-j_7r_5] [D_7(a_6)-D_6(a_7)] \end{aligned}$$

$$\begin{aligned} & +j_3r_3 [D_5(a_6)-D_6(a_5)] \\ & +j_3r_5 [D_6(a_3)-D_3(a_6)] \\ & +j_3r_6 [D_3(a_5)-D_5(a_3)] \end{aligned}$$

$$\begin{aligned} & [4] \\ & [+j_3r_6-j_6r_3] [D_5(a_4)-D_4(a_5)] \\ & [+j_5r_3-j_3r_5] [D_6(a_4)-D_4(a_6)] \\ & [+j_6r_5-j_5r_6] [D_3(a_4)-D_4(a_3)] \end{aligned}$$

$$\begin{aligned} & +j_4r_3 [D_5(a_6)-D_6(a_5)] \\ & +j_4r_5 [D_6(a_3)-D_3(a_6)] \\ & +j_4r_6 [D_3(a_5)-D_5(a_3)] \end{aligned}$$

$$\begin{aligned} & [5] \\ & +j_0r_3 [D_0(a_6)+D_6(a_0)] \\ & -j_0r_6 [D_0(a_3)+D_3(a_0)] \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_3(a_6)-D_6(a_3)]$$

$$\begin{aligned} & [+j_3r_1-j_1r_3] [D_6(a_1)-D_1(a_6)] \\ & [+j_3r_2-j_2r_3] [D_6(a_2)-D_2(a_6)] \\ & [+j_3r_4-j_4r_3] [D_6(a_4)-D_4(a_6)] \\ & [+j_3r_7-j_7r_3] [D_6(a_7)-D_7(a_6)] \end{aligned}$$

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$$\begin{aligned} & [+j_6r_1-j_1r_6] [D_1(a_3)-D_3(a_1)] \\ & [+j_6r_2-j_2r_6] [D_2(a_3)-D_3(a_2)] \\ & [+j_6r_4-j_4r_6] [D_4(a_3)-D_3(a_4)] \\ & [+j_6r_7-j_7r_6] [D_7(a_3)-D_3(a_7)] \end{aligned}$$

$$\begin{aligned} & +j_5r_3 [D_5(a_6)-D_6(a_5)] \\ & +j_5r_5 [D_6(a_3)-D_3(a_6)] \\ & +j_5r_6 [D_3(a_5)-D_5(a_3)] \end{aligned}$$

[6]

$$\begin{aligned} & +j_0r_5 [D_0(a_3)+D_3(a_0)] \\ & -j_0r_3 [D_0(a_5)+D_5(a_0)] \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_5(a_3)-D_3(a_5)]$$

$$\begin{aligned} & [+j_5r_1-j_1r_5] [D_3(a_1)-D_1(a_3)] \\ & [+j_5r_2-j_2r_5] [D_3(a_2)-D_2(a_3)] \\ & [+j_5r_4-j_4r_5] [D_3(a_4)-D_4(a_3)] \\ & [+j_5r_7-j_7r_5] [D_3(a_7)-D_7(a_3)] \\ & [+j_3r_1-j_1r_3] [D_1(a_5)-D_5(a_1)] \\ & [+j_3r_2-j_2r_3] [D_2(a_5)-D_5(a_2)] \\ & [+j_3r_4-j_4r_3] [D_4(a_5)-D_5(a_4)] \\ & [+j_3r_7-j_7r_3] [D_7(a_5)-D_5(a_7)] \end{aligned}$$

$$\begin{aligned} & +j_6r_3 [D_5(a_6)-D_6(a_5)] \\ & +j_6r_5 [D_6(a_3)-D_3(a_6)] \\ & +j_6r_6 [D_3(a_5)-D_5(a_3)] \end{aligned}$$

[7]

$$\begin{aligned} & [+j_3r_6-j_6r_3] [D_5(a_7)-D_7(a_5)] \\ & [+j_5r_3-j_3r_5] [D_6(a_7)-D_7(a_6)] \\ & [+j_6r_5-j_5r_6] [D_3(a_7)-D_7(a_3)] \end{aligned}$$

$$\begin{aligned} & +j_7r_3 [D_5(a_6)-D_6(a_5)] \\ & +j_7r_5 [D_6(a_3)-D_3(a_6)] \\ & +j_7r_6 [D_3(a_5)-D_5(a_3)] \end{aligned}$$

Variant $\frac{1}{2}$ (SL{541} + SR{541})

[2]

$$\begin{aligned} & [+j_3r_7-j_7r_3] [D_0(a_6)+D_6(a_0)] \\ & [+j_6r_3-j_3r_6] [D_0(a_7)+D_7(a_0)] \\ & [+j_7r_6-j_6r_7] [D_0(a_3)+D_3(a_0)] \end{aligned}$$

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$$\begin{aligned} &+j_0r_3 [D_7(a_6)-D_6(a_7)] \\ &+j_0r_6 [D_3(a_7)-D_7(a_3)] \\ &+j_0r_7 [D_6(a_3)-D_3(a_6)] \end{aligned}$$

$$\begin{aligned} [3] \\ &[+j_2r_6-j_6r_2] [D_0(a_7)+D_7(a_0)] \\ &[+j_6r_7-j_7r_6] [D_0(a_2)+D_2(a_0)] \\ &[+j_7r_2-j_2r_7] [D_0(a_6)+D_6(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_2 [D_6(a_7)-D_7(a_6)] \\ &+j_0r_6 [D_7(a_2)-D_2(a_7)] \\ &+j_0r_7 [D_2(a_6)-D_6(a_2)] \end{aligned}$$

$$\begin{aligned} [6] \\ &[+j_2r_7-j_7r_2] [D_0(a_3)+D_3(a_0)] \\ &[+j_3r_2-j_2r_3] [D_0(a_7)+D_7(a_0)] \\ &[+j_7r_3-j_3r_7] [D_0(a_2)+D_2(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_2 [D_7(a_3)-D_3(a_7)] \\ &+j_0r_3 [D_2(a_7)-D_7(a_2)] \\ &+j_0r_7 [D_3(a_2)-D_2(a_3)] \end{aligned}$$

$$\begin{aligned} [7] \\ &[+j_2r_3-j_3r_2] [D_0(a_6)+D_6(a_0)] \\ &[+j_3r_6-j_6r_3] [D_0(a_2)+D_2(a_0)] \\ &[+j_6r_2-j_2r_6] [D_0(a_3)+D_3(a_0)] \end{aligned}$$

$$\begin{aligned} &+j_0r_2 [D_3(a_6)-D_6(a_3)] \\ &+j_0r_3 [D_6(a_2)-D_2(a_6)] \\ &+j_0r_6 [D_2(a_3)-D_3(a_2)] \end{aligned}$$

Variant $\frac{1}{2}$ (SL{541} – SR{541})

$$\begin{aligned} [1] \\ &+j_0r_4 [D_0(a_5)+D_5(a_0)] \\ &-j_0r_5 [D_0(a_4)+D_4(a_0)] \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_5(a_4)-D_4(a_5)]$$

$$\begin{aligned} &[+j_4r_2-j_2r_4] [D_5(a_2)-D_2(a_5)] \\ &[+j_4r_3-j_3r_4] [D_5(a_3)-D_3(a_5)] \\ &[+j_4r_6-j_6r_4] [D_5(a_6)-D_6(a_5)] \\ &[+j_4r_7-j_7r_4] [D_5(a_7)-D_7(a_5)] \\ &[+j_5r_2-j_2r_5] [D_2(a_4)-D_4(a_2)] \end{aligned}$$

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$$\begin{aligned}
 & [+j_5r_3-j_3r_5] [D_3(a_4)-D_4(a_3)] \\
 & [+j_5r_6-j_6r_5] [D_6(a_4)-D_4(a_6)] \\
 & [+j_5r_7-j_7r_5] [D_7(a_4)-D_4(a_7)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_1r_1 [D_5(a_4)-D_4(a_5)] \\
 & +j_1r_4 [D_1(a_5)-D_5(a_1)] \\
 & +j_1r_5 [D_4(a_1)-D_1(a_4)]
 \end{aligned}$$

$$\begin{aligned}
 & [2] \\
 & [+j_1r_4-j_4r_1] [D_5(a_2)-D_2(a_5)] \\
 & [+j_4r_5-j_5r_4] [D_1(a_2)-D_2(a_1)] \\
 & [+j_5r_1-j_1r_5] [D_4(a_2)-D_2(a_4)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_2r_1 [D_5(a_4)-D_4(a_5)] \\
 & +j_2r_4 [D_1(a_5)-D_5(a_1)] \\
 & +j_2r_5 [D_4(a_1)-D_1(a_4)]
 \end{aligned}$$

$$\begin{aligned}
 & [3] \\
 & [+j_5r_1-j_1r_5] [D_4(a_3)-D_3(a_4)] \\
 & [+j_1r_4-j_4r_1] [D_5(a_3)-D_3(a_5)] \\
 & [+j_4r_5-j_5r_4] [D_1(a_3)-D_3(a_1)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_3r_1 [D_5(a_4)-D_4(a_5)] \\
 & +j_3r_4 [D_1(a_5)-D_5(a_1)] \\
 & +j_3r_5 [D_4(a_1)-D_1(a_4)]
 \end{aligned}$$

$$\begin{aligned}
 & [4] \\
 & +j_0r_5 [D_0(a_1)+D_1(a_0)] \\
 & -j_0r_1 [D_0(a_5)+D_5(a_0)]
 \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_5(a_1)-D_1(a_5)]$$

$$\begin{aligned}
 & [+j_5r_2-j_2r_5] [D_1(a_2)-D_2(a_1)] \\
 & [+j_5r_3-j_3r_5] [D_1(a_3)-D_3(a_1)] \\
 & [+j_5r_6-j_6r_5] [D_1(a_6)-D_6(a_1)] \\
 & [+j_5r_7-j_7r_5] [D_1(a_7)-D_7(a_1)] \\
 & [+j_1r_2-j_2r_1] [D_2(a_5)-D_5(a_2)] \\
 & [+j_1r_3-j_3r_1] [D_3(a_5)-D_5(a_3)] \\
 & [+j_1r_6-j_6r_1] [D_6(a_5)-D_5(a_6)] \\
 & [+j_1r_7-j_7r_1] [D_7(a_5)-D_5(a_7)]
 \end{aligned}$$

$$\begin{aligned}
 & +j_4r_1 [D_5(a_4)-D_4(a_5)] \\
 & +j_4r_4 [D_1(a_5)-D_5(a_1)] \\
 & +j_4r_5 [D_4(a_1)-D_1(a_4)]
 \end{aligned}$$

$$[5]$$

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$$+j_0r_1 [D_0(a_4)+D_4(a_0)]$$

$$-j_0r_4 [D_0(a_1)+D_1(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_1(a_4)-D_4(a_1)]$$

$$[+j_1r_2-j_2r_1] [D_4(a_2)-D_2(a_4)]$$

$$[+j_1r_3-j_3r_1] [D_4(a_3)-D_3(a_4)]$$

$$[+j_1r_6-j_6r_1] [D_4(a_6)-D_6(a_4)]$$

$$[+j_1r_7-j_7r_1] [D_4(a_7)-D_7(a_4)]$$

$$[+j_4r_2-j_2r_4] [D_2(a_1)-D_1(a_2)]$$

$$[+j_4r_3-j_3r_4] [D_3(a_1)-D_1(a_3)]$$

$$[+j_4r_6-j_6r_4] [D_6(a_1)-D_1(a_6)]$$

$$[+j_4r_7-j_7r_4] [D_7(a_1)-D_1(a_7)]$$

$$+j_5r_1 [D_5(a_4)-D_4(a_5)]$$

$$+j_5r_4 [D_1(a_5)-D_5(a_1)]$$

$$+j_5r_5 [D_4(a_1)-D_1(a_4)]$$

[6]

$$[+j_1r_4-j_4r_1] [D_5(a_6)-D_6(a_5)]$$

$$[+j_4r_5-j_5r_4] [D_1(a_6)-D_6(a_1)]$$

$$[+j_5r_1-j_1r_5] [D_4(a_6)-D_6(a_4)]$$

$$+j_6r_1 [D_5(a_4)-D_4(a_5)]$$

$$+j_6r_4 [D_1(a_5)-D_5(a_1)]$$

$$+j_6r_5 [D_4(a_1)-D_1(a_4)]$$

[7]

$$[+j_1r_4-j_4r_1] [D_5(a_7)-D_7(a_5)]$$

$$[+j_4r_5-j_5r_4] [D_1(a_7)-D_7(a_1)]$$

$$[+j_5r_1-j_1r_5] [D_4(a_7)-D_7(a_4)]$$

$$+j_7r_1 [D_5(a_4)-D_4(a_5)]$$

$$+j_7r_4 [D_1(a_5)-D_5(a_1)]$$

$$+j_7r_5 [D_4(a_1)-D_1(a_4)]$$

Variant $\frac{1}{2}$ (SL{642} + SR{642})

[1]

$$[+j_3r_7-j_7r_3] [D_0(a_5)+D_5(a_0)]$$

$$[+j_5r_3-j_3r_5] [D_0(a_7)+D_7(a_0)]$$

$$[+j_7r_5-j_5r_7] [D_0(a_3)+D_3(a_0)]$$

$$+j_0r_3 [D_7(a_5)-D_5(a_7)]$$

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$$+j_0r_5 [D_3(a_7)-D_7(a_3)]$$
$$+j_0r_7 [D_5(a_3)-D_3(a_5)]$$

$$[3]$$
$$[+j_1r_5-j_5r_1] [D_0(a_7)+D_7(a_0)]$$
$$[+j_5r_7-j_7r_5] [D_0(a_1)+D_1(a_0)]$$
$$[+j_7r_1-j_1r_7] [D_0(a_5)+D_5(a_0)]$$

$$+j_0r_1 [D_5(a_7)-D_7(a_5)]$$
$$+j_0r_5 [D_7(a_1)-D_1(a_7)]$$
$$+j_0r_7 [D_1(a_5)-D_5(a_1)]$$

$$[5]$$
$$[+j_1r_7-j_7r_1] [D_0(a_3)+D_3(a_0)]$$
$$[+j_3r_1-j_1r_3] [D_0(a_7)+D_7(a_0)]$$
$$[+j_7r_3-j_3r_7] [D_0(a_1)+D_1(a_0)]$$

$$+j_0r_1 [D_7(a_3)-D_3(a_7)]$$
$$+j_0r_3 [D_1(a_7)-D_7(a_1)]$$
$$+j_0r_7 [D_3(a_1)-D_1(a_3)]$$

$$[7]$$
$$[+j_1r_3-j_3r_1] [D_0(a_5)+D_5(a_0)]$$
$$[+j_3r_5-j_5r_3] [D_0(a_1)+D_1(a_0)]$$
$$[+j_5r_1-j_1r_5] [D_0(a_3)+D_3(a_0)]$$

$$+j_0r_1 [D_3(a_5)-D_5(a_3)]$$
$$+j_0r_3 [D_5(a_1)-D_1(a_5)]$$
$$+j_0r_5 [D_1(a_3)-D_3(a_1)]$$

Variant $1/2$ (SL{642} – SR{642})

$$[1]$$
$$[+j_2r_4-j_4r_2] [D_6(a_1)-D_1(a_6)]$$
$$[+j_4r_6-j_6r_4] [D_2(a_1)-D_1(a_2)]$$
$$[+j_6r_2-j_2r_6] [D_4(a_1)-D_1(a_4)]$$

$$+j_1r_2 [D_6(a_4)-D_4(a_6)]$$
$$+j_1r_4 [D_2(a_6)-D_6(a_2)]$$
$$+j_1r_6 [D_4(a_2)-D_2(a_4)]$$

$$[2]$$
$$+j_0r_4D_0(a_6)+j_0r_4D_6(a_0)$$
$$-j_0r_6D_0(a_4)-j_0r_6D_4(a_0)$$

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$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_4(a_6)-D_6(a_4)]$$

$$\begin{aligned} & [+j_4r_1-j_1r_4] [D_6(a_1)-D_1(a_6)] \\ & [+j_4r_3-j_3r_4] [D_6(a_3)-D_3(a_6)] \\ & [+j_4r_5-j_5r_4] [D_6(a_5)-D_5(a_6)] \\ & [+j_4r_7-j_7r_4] [D_6(a_7)-D_7(a_6)] \\ & [+j_6r_1-j_1r_6] [D_1(a_4)-D_4(a_1)] \\ & [+j_6r_3-j_3r_6] [D_3(a_4)-D_4(a_3)] \\ & [+j_6r_5-j_5r_6] [D_5(a_4)-D_4(a_5)] \\ & [+j_6r_7-j_7r_6] [D_7(a_4)-D_4(a_7)] \end{aligned}$$

$$\begin{aligned} & +j_2r_2 [D_6(a_4)-D_4(a_6)] \\ & +j_2r_4 [D_2(a_6)-D_6(a_2)] \\ & +j_2r_6 [D_4(a_2)-D_2(a_4)] \end{aligned}$$

[3]

$$\begin{aligned} & [+j_2r_4-j_4r_2] [D_6(a_3)-D_3(a_6)] \\ & [+j_4r_6-j_6r_4] [D_2(a_3)-D_3(a_2)] \\ & [+j_6r_2-j_2r_6] [D_4(a_3)-D_3(a_4)] \end{aligned}$$

$$\begin{aligned} & +j_3r_2 [D_6(a_4)-D_4(a_6)] \\ & +j_3r_4 [D_2(a_6)-D_6(a_2)] \\ & +j_3r_6 [D_4(a_2)-D_2(a_4)] \end{aligned}$$

[4]

$$\begin{aligned} & +j_0r_6 [D_0(a_2)+D_2(a_0)] \\ & -j_0r_2 [D_0(a_6)+D_6(a_0)] \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_6(a_2)-D_2(a_6)]$$

$$\begin{aligned} & [+j_6r_1-j_1r_6] [D_2(a_1)-D_1(a_2)] \\ & [+j_6r_3-j_3r_6] [D_2(a_3)-D_3(a_2)] \\ & [+j_6r_5-j_5r_6] [D_2(a_5)-D_5(a_2)] \\ & [+j_6r_7-j_7r_6] [D_2(a_7)-D_7(a_2)] \\ & [+j_2r_1-j_1r_2] [D_1(a_6)-D_6(a_1)] \\ & [+j_2r_3-j_3r_2] [D_3(a_6)-D_6(a_3)] \\ & [+j_2r_5-j_5r_2] [D_5(a_6)-D_6(a_5)] \\ & [+j_2r_7-j_7r_2] [D_7(a_6)-D_6(a_7)] \end{aligned}$$

$$\begin{aligned} & +j_4r_2 [D_6(a_4)-D_4(a_6)] \\ & +j_4r_4 [D_2(a_6)-D_6(a_2)] \\ & +j_4r_6 [D_4(a_2)-D_2(a_4)] \end{aligned}$$

[5]

$$[+j_2r_4-j_4r_2] [D_6(a_5)-D_5(a_6)]$$

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$$[+j_4r_6-j_6r_4] [D_2(a_5)-D_5(a_2)]$$
$$[+j_6r_2-j_2r_6] [D_4(a_5)-D_5(a_4)]$$

$$+j_5r_2 [D_6(a_4)-D_4(a_6)]$$
$$+j_5r_4 [D_2(a_6)-D_6(a_2)]$$
$$+j_5r_6 [D_4(a_2)-D_2(a_4)]$$

$$[6]$$
$$+j_0r_2 [D_0(a_4)+D_4(a_0)]$$
$$-j_0r_4 [D_0(a_2)+D_2(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_2(a_4)-D_4(a_2)]$$

$$[+j_2r_1-j_1r_2] [D_4(a_1)-D_1(a_4)]$$
$$[+j_2r_3-j_3r_2] [D_4(a_3)-D_3(a_4)]$$
$$[+j_2r_5-j_5r_2] [D_4(a_5)-D_5(a_4)]$$
$$[+j_2r_7-j_7r_2] [D_4(a_7)-D_7(a_4)]$$
$$[+j_4r_1-j_1r_4] [D_1(a_2)-D_2(a_1)]$$
$$[+j_4r_3-j_3r_4] [D_3(a_2)-D_2(a_3)]$$
$$[+j_4r_5-j_5r_4] [D_5(a_2)-D_2(a_5)]$$
$$[+j_4r_7-j_7r_4] [D_7(a_2)-D_2(a_7)]$$

$$+j_6r_2 [D_6(a_4)-D_4(a_6)]$$
$$+j_6r_4 [D_2(a_6)-D_6(a_2)]$$
$$+j_6r_6 [D_4(a_2)-D_2(a_4)]$$

$$[7]$$
$$[+j_2r_4-j_4r_2] [D_6(a_7)-D_7(a_6)]$$
$$[+j_4r_6-j_6r_4] [D_2(a_7)-D_7(a_2)]$$
$$[+j_6r_2-j_2r_6] [D_4(a_7)-D_7(a_4)]$$

$$+j_7r_2 [D_6(a_4)-D_4(a_6)]$$
$$+j_7r_4 [D_2(a_6)-D_6(a_2)]$$
$$+j_7r_6 [D_4(a_2)-D_2(a_4)]$$

Variant $\frac{1}{2}$ (SL{743} + SR{743})

$$[1]$$
$$[+j_2r_6-j_6r_2] [D_0(a_5)+D_5(a_0)]$$
$$[+j_5r_2-j_2r_5] [D_0(a_6)+D_6(a_0)]$$
$$[+j_6r_5-j_5r_6] [D_0(a_2)+D_2(a_0)]$$

$$+j_0r_2 [D_6(a_5)-D_5(a_6)]$$
$$+j_0r_5 [D_2(a_6)-D_6(a_2)]$$

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$$+j_0r_6 [D_5(a_2)-D_2(a_5)]$$

[2]

$$[+j_1r_5-j_5r_1] [D_0(a_6)+D_6(a_0)]$$

$$[+j_5r_6-j_6r_5] [D_0(a_1)+D_1(a_0)]$$

$$[+j_6r_1-j_1r_6] [D_0(a_5)+D_5(a_0)]$$

$$+j_0r_1 [D_5(a_6)-D_6(a_5)]$$

$$+j_0r_5 [D_6(a_1)-D_1(a_6)]$$

$$+j_0r_6 [D_1(a_5)-D_5(a_1)]$$

[5]

$$[+j_1r_6-j_6r_1] [D_0(a_2)+D_2(a_0)]$$

$$[+j_2r_1-j_1r_2] [D_0(a_6)+D_6(a_0)]$$

$$[+j_6r_2-j_2r_6] [D_0(a_1)+D_1(a_0)]$$

$$+j_0r_1 [D_6(a_2)-D_2(a_6)]$$

$$+j_0r_2 [D_1(a_6)-D_6(a_1)]$$

$$+j_0r_6 [D_2(a_1)-D_1(a_2)]$$

[6]

$$[+j_1r_2-j_2r_1] [D_0(a_5)+D_5(a_0)]$$

$$[+j_2r_5-j_5r_2] [D_0(a_1)+D_1(a_0)]$$

$$[+j_5r_1-j_1r_5] [D_0(a_2)+D_2(a_0)]$$

$$+j_0r_1 [D_2(a_5)-D_5(a_2)]$$

$$+j_0r_2 [D_5(a_1)-D_1(a_5)]$$

$$+j_0r_5 [D_1(a_2)-D_2(a_1)]$$

Variant $\frac{1}{2}$ (SL{743} – SR{743})

[1]

$$[+j_3r_4-j_4r_3] [D_7(a_1)-D_1(a_7)]$$

$$[+j_4r_7-j_7r_4] [D_3(a_1)-D_1(a_3)]$$

$$[+j_7r_3-j_3r_7] [D_4(a_1)-D_1(a_4)]$$

$$+j_1r_3 [D_7(a_4)-D_4(a_7)]$$

$$+j_1r_4 [D_3(a_7)-D_7(a_3)]$$

$$+j_1r_7 [D_4(a_3)-D_3(a_4)]$$

[2]

$$[+j_3r_4-j_4r_3] [D_7(a_2)-D_2(a_7)]$$

$$[+j_4r_7-j_7r_4] [D_3(a_2)-D_2(a_3)]$$

$$[+j_7r_3-j_3r_7] [D_4(a_2)-D_2(a_4)]$$

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$$\begin{aligned}
 &+j_2r_3 [D_7(a_4)-D_4(a_7)] \\
 &+j_2r_4 [D_3(a_7)-D_7(a_3)] \\
 &+j_2r_7 [D_4(a_3)-D_3(a_4)]
 \end{aligned}$$

[3]

$$\begin{aligned}
 &+j_0r_4 [D_0(a_7)+D_7(a_0)] \\
 &-j_0r_7 [D_0(a_4)+D_4(a_0)]
 \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_4(a_7)-D_7(a_4)]$$

$$\begin{aligned}
 &[+j_4r_1-j_1r_4] [D_7(a_1)-D_1(a_7)] \\
 &[+j_4r_2-j_2r_4] [D_7(a_2)-D_2(a_7)] \\
 &[+j_4r_5-j_5r_4] [D_7(a_5)-D_5(a_7)] \\
 &[+j_4r_6-j_6r_4] [D_7(a_6)-D_6(a_7)] \\
 &[+j_7r_1-j_1r_7] [D_1(a_4)-D_4(a_1)] \\
 &[+j_7r_2-j_2r_7] [D_2(a_4)-D_4(a_2)] \\
 &[+j_7r_5-j_5r_7] [D_5(a_4)-D_4(a_5)] \\
 &[+j_7r_6-j_6r_7] [D_6(a_4)-D_4(a_6)]
 \end{aligned}$$

$$\begin{aligned}
 &+j_3r_3 [D_7(a_4)-D_4(a_7)] \\
 &+j_3r_4 [D_3(a_7)-D_7(a_3)] \\
 &+j_3r_7 [D_4(a_3)-D_3(a_4)]
 \end{aligned}$$

[4]

$$\begin{aligned}
 &+j_0r_7 [D_0(a_3)+D_3(a_0)] \\
 &-j_0r_3 [D_0(a_7)+D_7(a_0)]
 \end{aligned}$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_7(a_3)-D_3(a_7)]$$

$$\begin{aligned}
 &[+j_7r_1-j_1r_7] [D_3(a_1)-D_1(a_3)] \\
 &[+j_7r_2-j_2r_7] [D_3(a_2)-D_2(a_3)] \\
 &[+j_7r_5-j_5r_7] [D_3(a_5)-D_5(a_3)] \\
 &[+j_7r_6-j_6r_7] [D_3(a_6)-D_6(a_3)] \\
 &[+j_3r_1-j_1r_3] [D_1(a_7)-D_7(a_1)] \\
 &[+j_3r_2-j_2r_3] [D_2(a_7)-D_7(a_2)] \\
 &[+j_3r_5-j_5r_3] [D_5(a_7)-D_7(a_5)] \\
 &[+j_3r_6-j_6r_3] [D_6(a_7)-D_7(a_6)]
 \end{aligned}$$

$$\begin{aligned}
 &+j_4r_3 [D_7(a_4)-D_4(a_7)] \\
 &+j_4r_4 [D_3(a_7)-D_7(a_3)] \\
 &+j_4r_7 [D_4(a_3)-D_3(a_4)]
 \end{aligned}$$

[5]

$$\begin{aligned}
 &[+j_3r_4-j_4r_3] [D_7(a_5)-D_5(a_7)] \\
 &[+j_4r_7-j_7r_4] [D_3(a_5)-D_5(a_3)]
 \end{aligned}$$

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$$[+j_7r_3-j_3r_7] [D_4(a_5)-D_5(a_4)]$$

$$+j_5r_3 [D_7(a_4)-D_4(a_7)]$$

$$+j_5r_4 [D_3(a_7)-D_7(a_3)]$$

$$+j_5r_7 [D_4(a_3)-D_3(a_4)]$$

[6]

$$[+j_3r_4-j_4r_3] [D_7(a_6)-D_6(a_7)]$$

$$[+j_4r_7-j_7r_4] [D_3(a_6)-D_6(a_3)]$$

$$[+j_7r_3-j_3r_7] [D_4(a_6)-D_6(a_4)]$$

$$+j_6r_3 [D_7(a_4)-D_4(a_7)]$$

$$+j_6r_4 [D_3(a_7)-D_7(a_3)]$$

$$+j_6r_7 [D_4(a_3)-D_3(a_4)]$$

[7]

$$+j_0r_3 [D_0(a_4)+D_4(a_0)]$$

$$-j_0r_4 [D_0(a_3)+D_3(a_0)]$$

$$[+j_1r_1+j_2r_2+j_3r_3+j_4r_4+j_5r_5+j_6r_6+j_7r_7] [D_3(a_4)-D_4(a_3)]$$

$$[+j_3r_1-j_1r_3] [D_4(a_1)-D_1(a_4)]$$

$$[+j_3r_2-j_2r_3] [D_4(a_2)-D_2(a_4)]$$

$$[+j_3r_5-j_5r_3] [D_4(a_5)-D_5(a_4)]$$

$$[+j_3r_6-j_6r_3] [D_4(a_6)-D_6(a_4)]$$

$$[+j_4r_1-j_1r_4] [D_1(a_3)-D_3(a_1)]$$

$$[+j_4r_2-j_2r_4] [D_2(a_3)-D_3(a_2)]$$

$$[+j_4r_5-j_5r_4] [D_5(a_3)-D_3(a_5)]$$

$$[+j_4r_6-j_6r_4] [D_6(a_3)-D_3(a_6)]$$

$$+j_7r_3 [D_7(a_4)-D_4(a_7)]$$

$$+j_7r_4 [D_3(a_7)-D_7(a_3)]$$

$$+j_7r_7 [D_4(a_3)-D_3(a_4)]$$

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