When the associator

½ [a*(b*c) – (a*b)*c]

is cranked through the Octonion Variance Sieve Process[1], the algebraic invariant portion of the result is identically zero. This does not imply the algebraic invariant portion of any Octonion product string is fully associative.

Take for instance the algebraic invariant unit product

$u_3 [(u_5 u_3) (u_1 u_5)]$

This is a unit product string that involves the following 4 unit multiplication permutations

$(u_1\ u_5\ u_4)$
$(u_5\ u_3\ u_6)$
$(u_6\ u_4\ u_2)$
$(u_1\ u_2\ u_3)$

These are the 4 permutations that do not include the unit 7. When any valid[1] Octonion algebra change is made, either all, none, or two of these permutations will be negated. This leaves the resultant sign unchanged for any possible algebra.

If associative modification to the product order is attempted, say right to left, the 4 permutations involved are now

$(u_1\ u_5\ u_4)$
$(u_3\ u_7\ u_4)$
$(u_5\ u_7\ u_2)$
$(u_1\ u_2\ u_3)$

These are the 4 permutations that do not include the unit 6, not the same grouping. The result is the same unit, but the sign changes. Associativity is broken, even though the result is again an invariant form.

The variant results of the Octonion Varaince Sieve Process are given below in the following minimum distances.

For minimum distance 1: ½[SL(123)+SR(321)]

```
[4]-a5b6c7+a5b7c6+a6b5c7-a6b7c5-a7b5c6+a7b6c5
[5]+a4b6c7-a4b7c6-a6b4c7+a6b7c4+a7b4c6-a7b6c4
[6]-a4b5c7+a4b7c5+a5b4c7-a5b7c4-a7b4c5+a7b5c4
[7]+a4b5c6-a4b6c5-a5b4c6+a5b6c4+a6b4c5-a6b5c4
```

For minimum distance 1: ½[SL(123)-SR(321)]


```
Null
```

For minimum distance 2: ½[SL(761)+SR(167)]

```
[2]+a3b4c5-a3b5c4-a4b3c5+a4b5c3+a5b3c4-a5b4c3
[3]-a2b4c5+a2b5c4+a4b2c5-a4b5c2-a5b2c4+a5b4c2
[4]+a2b3c5-a2b5c3-a3b2c5+a3b5c2+a5b2c3-a5b3c2
[5]-a2b3c4+a2b4c3+a3b2c4-a3b4c2-a4b2c3+a4b3c2
```

For minimum distance 2: ½[SL(761)-SR(167)]

```
Null
```

For minimum distance 3: ½[SL(572)+SR(275)]

```
[1]-a3b4c6+a3b6c4+a4b3c6-a4b6c3-a6b3c4+a6b4c3
[3]+a1b4c6-a1b6c4-a4b1c6+a4b6c1+a6b1c4-a6b4c1
[4]-a1b3c6+a1b6c3+a3b1c6-a3b6c1-a6b1c3+a6b3c1
[6]+a1b3c4-a1b4c3-a3b1c4+a3b4c1+a4b1c3-a4b3c1
```

For minimum distance 3: ½[SL(572)-SR(275)]

```
Null
```

For minimum distance 4: ½[SL(653)+SR(356)]

```
[1]+a2b4c7-a2b7c4-a4b2c7+a4b7c2+a7b2c4-a7b4c2
[2]-a1b4c7+a1b7c4+a4b1c7-a4b7c1-a7b1c4+a7b4c1
[4]+a1b2c7-a1b7c2-a2b1c7+a2b7c1+a7b1c2-a7b2c1
[7]-a1b2c4+a1b4c2+a2b1c4-a2b4c1-a4b1c2+a4b2c1
```

For minimum distance 4: ½[SL(653)-SR(356)]

```
Null
```

For minimum distance 5: ½[SL(145)+SR(541)]

```
[2]-a3b6c7+a3b7c6+a6b3c7-a6b7c3-a7b3c6+a7b6c3
[3]+a2b6c7-a2b7c6-a6b2c7+a6b7c2+a7b2c6-a7b6c2
[6]-a2b3c7+a2b7c3+a3b2c7-a3b7c2-a7b2c3+a7b3c2
[7]+a2b3c6-a2b6c3-a3b2c6+a3b6c2+a6b2c3-a6b3c2
```

For minimum distance 5: ½[SL(145)-SR(541)]

```
Null
```

For minimum distance 6: ½[SL(246)+SR(642)]

```
[1]-a3b5c7+a3b7c5+a5b3c7-a5b7c3-a7b3c5+a7b5c3
[3]+a1b5c7-a1b7c5-a5b1c7+a5b7c1+a7b1c5-a7b5c1
[5]-a1b3c7+a1b7c3+a3b1c7-a3b7c1-a7b1c3+a7b3c1
[7]+a1b3c5-a1b5c3-a3b1c5+a3b5c1+a5b1c3-a5b3c1
```

For minimum distance 6: ½[SL(246)-SR(642)]

Null

For minimum distance 7: ½[SL(347)+SR(743)]

```
[1]-a2b5c6+a2b6c5+a5b2c6-a5b6c2-a6b2c5+a6b5c2
[2]+a1b5c6-a1b6c5-a5b1c6+a5b6c1+a6b1c5-a6b5c1
[5]-a1b2c6+a1b6c2+a2b1c6-a2b6c1-a6b1c2+a6b2c1
[6]+a1b2c5-a1b5c2-a2b1c5+a2b5c1+a5b1c2-a5b2c1
```

For minimum distance 7: ½[SL(347)-SR(743)]

Null

Richard Lockyer
June 28, 2008

Bibliography

[1] © 2008 Richard Lockyer
http://www.octospace.com/files/Octonion_Algebra_and_its_Connection_to_Physics.pdf