16 Octonion Algebras – 30 Aliases – 480 Representations

The multiplication rules for Octonion Algebra basis elements define the bilinear operation $\mathbf{e}_r * \mathbf{e}_s = +/-\mathbf{e}_t$ for all r,s. For all possible Octonion Algebras and all basis set representations, scalar multiplication (r and/or s zero) as well as all products where r = s are singularly defined. Variation between the algebras is only possible within the rule sets where r,s and t are not zero and are not equal.

These rule sets are succinctly represented by a set of seven cyclic permutation triplets. For a single valid set of seven triplets, variations in the order of basis elements within the seven permutations, and hence the algebra's multiplication rules, must preserve both the normed and alternative algebra properties. Either will restrict things to the same sixteen possibilities. I have covered this at length already [1][2][3][4].

The focus of this paper is to explore the number of different sets of seven basis element triplets possible without consideration on the multiplication rules by which particular Octonion Algebras are defined. The triplets will only represent seven sets of three basis elements.

There are rules each possible seven triplet set must abide by in order to cover all multiplications where r, s and t above are unequal and non-zero.

(1) Each non-scalar basis element appears individually in three separate triplets.
(2) Each non-scalar basis element appears only once in a triplet with each of the other non-scalar basis elements.

Enumerate the basis elements as $\mathbf{e}_0$ $\mathbf{e}_g$ $\mathbf{e}_f$ $\mathbf{e}_e$ $\mathbf{e}_d$ $\mathbf{e}_c$ $\mathbf{e}_b$ $\mathbf{e}_a$. By (1) and (2), all possible choices of seven triplets will include the following partial assignments:

{ - $\mathbf{e}_f$ $\mathbf{e}_g$ }
{ - - $\mathbf{e}_g$ }
{ - - $\mathbf{e}_g$ }

We may freely choose the single unassigned basis element in the first triplet from one of the five remaining non-scalar basis elements. This starts our count of representations at five. For demonstration purposes, I will choose $\mathbf{e}_e$. Next examine the possibilities for the four unassigned basis elements in the triplets

{ $\mathbf{e}_e$ $\mathbf{e}_f$ $\mathbf{e}_g$ }
{ - - $\mathbf{e}_g$ }
{ - - $\mathbf{e}_g$ }

By (1) and (2) above, there are only three choices, they are

{ $\mathbf{e}_a$ $\mathbf{e}_b$ - }	{ $\mathbf{e}_a$ $\mathbf{e}_c$ - }	{ $\mathbf{e}_a$ $\mathbf{e}_d$ - }
{ $\mathbf{e}_c$ $\mathbf{e}_d$ - }	{ $\mathbf{e}_b$ $\mathbf{e}_d$ - }	{ $\mathbf{e}_b$ $\mathbf{e}_c$ - }

The count of possible representations is now partially 5*3 = 15. Again I will pick one of the three for demonstration purposes:

{ $\mathbf{e}_e\ \mathbf{e}_f\ \mathbf{e}_g$ }
{ $\mathbf{e}_a\ \mathbf{e}_b\ \mathbf{e}_g$ }
{ $\mathbf{e}_c\ \mathbf{e}_d\ \mathbf{e}_g$ }

Now $\mathbf{e}_e$ and $\mathbf{e}_f$ appear only in a single triplet. By (1) above, each must appear in a total of three, and since they already appear together, by (2) the two additional triplets for each must not include the other, so both appear twice in the remaining four triplets to assign. We may then specify additively the form the remaining four triplets must take:

{ $\mathbf{e}_e\ \mathbf{e}_f\ \mathbf{e}_g$ }
{ $\mathbf{e}_a\ \mathbf{e}_b\ \mathbf{e}_g$ }
{ $\mathbf{e}_c\ \mathbf{e}_d\ \mathbf{e}_g$ }
{ - - $\mathbf{e}_f$ }
{ - - $\mathbf{e}_e$ }
{ - - $\mathbf{e}_e$ }
{ - - $\mathbf{e}_f$ }

Here, basis elements $\mathbf{e}_e\ \mathbf{e}_f\ \mathbf{e}_g$ are fully committed so can't appear again, and basis elements $\mathbf{e}_a\ \mathbf{e}_b\ \mathbf{e}_c\ \mathbf{e}_d$ must each appear twice in the eight uncommitted positions. The six possible pairings of these four basis elements must be restricted to four due to the two pairings already used in the second and third triplet. The four pairings will then be:

{ $\mathbf{e}_a\ \mathbf{e}_c$ - } { $\mathbf{e}_a\ \mathbf{e}_d$ - } { $\mathbf{e}_b\ \mathbf{e}_c$ - } { $\mathbf{e}_b\ \mathbf{e}_d$ - }

By (2), there are only two ways these may be applied to our uncommitted positions:

{ $\mathbf{e}_e\ \mathbf{e}_f\ \mathbf{e}_g$ }	{ $\mathbf{e}_e\ \mathbf{e}_f\ \mathbf{e}_g$ }
{ $\mathbf{e}_a\ \mathbf{e}_b\ \mathbf{e}_g$ }	{ $\mathbf{e}_a\ \mathbf{e}_b\ \mathbf{e}_g$ }
{ $\mathbf{e}_c\ \mathbf{e}_d\ \mathbf{e}_g$ }	{ $\mathbf{e}_c\ \mathbf{e}_d\ \mathbf{e}_g$ }
{ $\mathbf{e}_a\ \mathbf{e}_c\ \mathbf{e}_f$ }	{ $\mathbf{e}_a\ \mathbf{e}_c\ \mathbf{e}_e$ }
{ $\mathbf{e}_a\ \mathbf{e}_d\ \mathbf{e}_e$ }	{ $\mathbf{e}_a\ \mathbf{e}_d\ \mathbf{e}_f$ }
{ $\mathbf{e}_b\ \mathbf{e}_c\ \mathbf{e}_e$ }	{ $\mathbf{e}_b\ \mathbf{e}_c\ \mathbf{e}_f$ }
{ $\mathbf{e}_b\ \mathbf{e}_d\ \mathbf{e}_f$ }	{ $\mathbf{e}_b\ \mathbf{e}_d\ \mathbf{e}_e$ }

We are done with all possibilities, and the number has been shown to be 5*3*2 = 30.

There is nothing of an algebraic defining nature between any of these thirty choices. They are simply different mappings of arbitrary basis element names to triplets. Any one will do for starting the definition of all sixteen Octonion Algebras, and all others are simply aliases. Together, they yield 16*30 = 480 Representations. I have provided all thirty aliases in what follows.

The Thirty Alias Representations

$\{ \mathbf{e}_e \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_e \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_e \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_e \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_e \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_e \mathbf{e}_f \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_g \}$
$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_e \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_e \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_e \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_f \}$
$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_e \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_e \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_e \}$
$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_e \}$	$\{ \mathbf{e}_c \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_c \mathbf{e}_b \mathbf{e}_e \}$	$\{ \mathbf{e}_d \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_b \mathbf{e}_e \}$
$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_e \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_e \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_c \mathbf{e}_e \}$	$\{ \mathbf{e}_d \mathbf{e}_c \mathbf{e}_f \}$

$\{ \mathbf{e}_d \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_f \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_g \}$
$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_d \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_d \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_d \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_f \}$
$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_d \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_d \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_d \}$
$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_d \}$	$\{ \mathbf{e}_c \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_c \mathbf{e}_b \mathbf{e}_d \}$	$\{ \mathbf{e}_e \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_b \mathbf{e}_d \}$
$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_d \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_d \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_c \mathbf{e}_d \}$	$\{ \mathbf{e}_e \mathbf{e}_c \mathbf{e}_f \}$

$\{ \mathbf{e}_c \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_f \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_g \}$
$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_c \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_c \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_c \}$	$\{ \mathbf{e}_a \mathbf{e}_b \mathbf{e}_f \}$
$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_c \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_c \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_c \}$
$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_c \}$	$\{ \mathbf{e}_d \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_b \mathbf{e}_c \}$	$\{ \mathbf{e}_e \mathbf{e}_b \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_b \mathbf{e}_c \}$
$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_c \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_c \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_d \mathbf{e}_c \}$	$\{ \mathbf{e}_e \mathbf{e}_d \mathbf{e}_f \}$

$\{ \mathbf{e}_b \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_f \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_g \}$
$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_g \}$
$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_b \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_b \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_b \}$	$\{ \mathbf{e}_a \mathbf{e}_c \mathbf{e}_f \}$
$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_b \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_e \mathbf{e}_b \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_a \mathbf{e}_d \mathbf{e}_b \}$
$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_b \}$	$\{ \mathbf{e}_d \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_c \mathbf{e}_b \}$	$\{ \mathbf{e}_e \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_c \mathbf{e}_b \}$
$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_b \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_b \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_d \mathbf{e}_b \}$	$\{ \mathbf{e}_e \mathbf{e}_d \mathbf{e}_f \}$

$\{ \mathbf{e}_a \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_f \mathbf{e}_g \}$	$\{ \mathbf{e}_a \mathbf{e}_f \mathbf{e}_g \}$
$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_g \}$
$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_g \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_g \}$
$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_a \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_a \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_a \}$	$\{ \mathbf{e}_b \mathbf{e}_c \mathbf{e}_f \}$
$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_a \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_e \mathbf{e}_a \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_b \mathbf{e}_d \mathbf{e}_a \}$
$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_f \}$	$\{ \mathbf{e}_c \mathbf{e}_d \mathbf{e}_a \}$	$\{ \mathbf{e}_d \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_c \mathbf{e}_a \}$	$\{ \mathbf{e}_e \mathbf{e}_c \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_c \mathbf{e}_a \}$
$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_a \}$	$\{ \mathbf{e}_c \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_a \}$	$\{ \mathbf{e}_d \mathbf{e}_e \mathbf{e}_f \}$	$\{ \mathbf{e}_e \mathbf{e}_d \mathbf{e}_a \}$	$\{ \mathbf{e}_e \mathbf{e}_d \mathbf{e}_f \}$

One tidbit of perhaps interesting, maybe expected information on these thirty choices is the following.

Pick any representation and any two basis elements within. Exchange their positions within the triplets. You will find the new set of seven triplets at another position.

References

[1] (2008) R. Lockyer, "Octonion Algebra and its Connection to Physics"
http://www.octospace.com/files/Octonion_Algebra_and_its_Connection_to_Physics.pdf

[2] sci.physics post, R. Lockyer, "Why Octonions 2: A Full Description of the Algebra"
http://groups.google.com/group/sci.physics/browse_thread/thread/8e026683e7eb1149/53c1e9e00f4d5fe1?q
August 8, 2008

[3] (2010) R. Lockyer, "Hadamard Matrix Connection to Octonion Algebras"
http://www.octospace.com/files/Hadamard_Matrix_Connection_to_Octonion_Algebra.pdf

[4] (2010) R. Lockyer "Quaternion Doubling for Right and Left Octonion Algebra"
http://www.octospace.com/files/Quaternion_Doubling_for_all_Octonion_Algebras.pdf

